

The Surprisingly Small Impact of Asset Growth on Expected Alpha

Warning: Optimal portfolio management required.

Ronald N. Kahn and J. Scott Shaffer

On August 27, 1997, Fidelity Investments announced it was closing its flagship Magellan Fund to new investors. The fund had underperformed the S&P 500 index in 1994, 1995, and 1996, and was underperforming year-to-date in 1997. During that same period, assets had nearly doubled, growing from just under \$32 billion to over \$60 billion.

According to *European Business News* at the time:

The move represents a bid . . . to quash criticism that the fund had grown too big to be managed effectively. Some say [portfolio manager Jeffrey] Vinik's 30% shift into cash and bonds in 1996 was a direct result of the fund's extreme size and managers' difficulty finding places to invest. Under the fund's new manager, the fund is again fully invested, but some critics say it now more closely resembles an index fund.

In somewhat conflicting observations, commentators also noted both that this move would have little impact on Magellan's asset growth, given the fund's substantial inflows from existing clients, and that this could increase Fidelity's profits if it led (through slower inflows) to better investment returns, because fees would rise substantially if the fund outperformed the S&P 500.

Was the Magellan Fund out of capacity in August 1997? Did its growth in assets lead to its underperfor-

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mance? More generally, what is capacity, and how do we determine it?

Managers, sponsors, and consultants all have an interest in understanding capacity. Managers have the means to analyze and monitor capacity in considerable detail. Sponsors and consultants can't achieve that same level of detail, but can monitor products for warning signs of capacity issues.

We propose a framework for characterizing capacity, and then apply it to an example product. We discuss how products should adjust as assets grow, provide some guidelines for monitoring capacity, and suggest ways to possibly increase capacity. While this framework proves quite useful, we also consider its important real-world shortcomings, which can lead it to overestimate capacity. Ultimately, those real-world details can limit capacity.

FRAMEWORK

We have encountered many approaches to analyzing capacity. At one end, there are ad hoc pronouncements (e.g., capacity is 1% of market size) or general observations (e.g., "Almost every active equity manager we have come across that has gone past 2% of market cap has underperformed over the period since then.")¹ While these have the benefit of concreteness—and few products exceed such limits—they surely can't apply to everything from high-turnover statistical arbitrage to Warren Buffett's very long investment horizon.

At the other end of the spectrum, we have also seen much more detailed analyses, typically applied in backtests. Researchers look at the simulated historical performance of an investment product as they vary its assets under management. The performance tends to degrade with increasing assets, because trading costs increase. Either the fund spends more on trading, or it is less able to implement good ideas (or both).

This approach requires forecasts of trading costs that depend on trade size (i.e., trading 100,000 shares costs more in percentage terms than trading 100 shares). But this approach depends too heavily on historical performance. If an idea performed extremely well over a particular period (perhaps before more investors started using the same idea), studies based only on that period may erroneously indicate extremely high capacity.

The academic finance literature has addressed this issue with regard to market anomalies. First, can investors exploit these anomalies after costs? Second, are

costs the reason why such anomalies still remain? Early researchers looked only at explicit costs (commissions and spreads), and so could address only whether investors could exploit these anomalies at all. More recent work has included market impact, which allows the estimation of capacity for these anomalies. All these academic studies mainly build simplistic portfolios—long the top decile and short the bottom decile, with stocks equal- or capitalization-weighted within deciles—without regard to costs.

Two academic efforts include Korajczyk and Sadka [2004] and Chen, Stanzl, and Watanabe [2002]. Korajczyk and Sadka [2004] look at the returns to momentum investing after trading costs, and observe the asset levels at which such after-cost returns drop to zero. Chen, Stanzl, and Watanabe [2002] similarly examine size, book-to-market, and momentum strategies in the presence of trading costs. Korajczyk and Sadka take the additional step of analyzing a third portfolio construction approach, with positions dependent on market cap *and* volume, but even they do not adjust position sizes or turnover as assets increase.

Perold and Salomon [1991] provide perhaps the earliest analysis. They define capacity according to maximizing total dollars of alpha. They then estimate capacity top-down using simple assumptions of average alpha, linear market impact, and a 100-position portfolio with 100% annual turnover. They work with reasonable estimates of these quantities, and do not forecast alphas or transaction costs for specific stocks.

Our approach provides detailed results, as well as sensitivity analysis, without depending too much on historical performance. It explicitly optimizes portfolio construction and trading for the asset level involved.

This approach analyzes capacity using expected active returns net of costs. In simple terms, how many of our ideas can we implement in the portfolio to deliver returns to investors? For any given asset level, we forecast the expected active return we can implement in steady state, given trading costs. We then subtract the expected costs, and compare that forecast active return net of costs to the performance our clients expect.

We have applied this approach to a wide range of products—from low-risk long-only to high-risk long-short—and believe it has broad applicability. Because of its detailed input requirements, this analysis is available only to managers analyzing their own products, but the general results and intuition can also help sponsors and consultants.

Alpha Net of Costs

Following the approach of Grinold and Kahn [2000a], we assume we are given a performance benchmark. (For a long-short product, the benchmark may be cash.) We define active return, θ , as the difference between the return to our portfolio and the return to the benchmark:²

$$\begin{aligned}\theta &= r_p - r_B \\ &= (\mathbf{h}_p - \mathbf{h}_B)^T \cdot \mathbf{r}\end{aligned}\quad (1)$$

where \mathbf{h} refers to the holdings in the portfolio or benchmark.

The active return fluctuates. We define alpha, α , as the expected active return, and omega, ω , as the active risk (i.e., the standard deviation of the active return):

$$\begin{aligned}\alpha &= E\{\theta\} \\ \omega &= StDev\{\theta\}\end{aligned}\quad (2)$$

We forecast active returns and risks for individual assets and portfolios.

Finally, we define the *information ratio* as the ratio of forecast active return to active risk:

$$IR = \frac{\alpha}{\omega}\quad (3)$$

The model for alpha net of cost is:

$$\alpha_{net} = \alpha_{gross} - \tau \cdot tc(A, \tau)\quad (4)$$

where τ measures the annual turnover, tc measures average trading costs, and A measures the asset level. The term τtc measures the annual trading costs at the fund level. For example, if trades cost 1% on average, and the fund experiences 50% annual turnover, trading costs the fund 0.50% per year.

We expect average trading costs to depend on asset level and turnover. As the dollar volume of trading increases, we expect trading costs to increase. We model the ex ante gross alpha as:

$$\alpha_{gross} = IR_{int} \cdot \omega \cdot e(\tau)\quad (5)$$

The gross alpha we implement depends on three quantities: the intrinsic information ratio, IR_{int} , the active risk level, ω , and our efficiency of implementation, e .³

The intrinsic information ratio is the ratio of expected active return to active risk we could achieve in the absence of all constraints and costs. It depends only on our asset-level alphas, and the asset-by-asset covariance matrix, \mathbf{V} :⁴

$$IR^2 = \alpha^T \cdot \mathbf{V}^{-1} \cdot \alpha\quad (6)$$

The efficiency measures how much of that intrinsic information ratio makes it into the portfolio, after constraints and costs. Given the actual portfolio, \mathbf{h}_p , which reflects the impact of constraints and costs, we can forecast its alpha, α_p (based on its holdings and our asset-level alphas), and its risk, ω_p :

$$\alpha_p = \mathbf{h}_p^T \cdot \alpha\quad (7)$$

$$\omega_p^2 = (\mathbf{h}_p - \mathbf{h}_B)^T \cdot \mathbf{V} \cdot (\mathbf{h}_p - \mathbf{h}_B)\quad (8)$$

We define efficiency as:

$$IR_p \equiv \frac{\alpha_p}{\omega_p} = e(\tau) \cdot IR_{int}\quad (9)$$

The maximum efficiency is 100%. We expect efficiency to increase with turnover, up to a maximum set by constraints. At very high turnover, we overcome efficiency drag due to costs. Of course, after costs, very high turnover will be suboptimal.

So overall, we define alpha net of costs as:

$$\alpha_{net} = IR_{int} \cdot \omega \cdot e(\tau) - \tau \cdot tc(A, \tau)\quad (10)$$

How do we estimate this quantity? We can freely specify the intrinsic information ratio, IR_{int} , depending on our assessment of the power of our investment ideas looking forward. The value we achieved historically will influence this forward view, but we can forecast this quantity even without a track record.

The active risk level is typically a specified characteristic of the product.

The greater challenges are to estimate efficiency, turnover, and average transaction costs. Here we will rely on backtests, but we will not use simulated investment

returns. Rather, we use only the efficiency, turnover, and cost estimates.

We approach this by first estimating efficiency and average transaction costs as functions of asset levels and turnover. We can then choose the turnover level that maximizes alpha net of costs, i.e., Equation (10). A maximum exists because, as we will demonstrate, efficiency initially rises faster than costs, but then costs catch up.

Here is one straightforward approach. We run a series of backtests over, e.g., three years of monthly data. For each backtest, we choose an asset level. We also scale our transaction costs. As we scale costs down, turnover rises. Note that we are scaling the costs as a means of varying turnover. For most of these runs, costs are artificially too high or too low. But our only goal here is to estimate efficiency as a function of turnover. We will later correctly account for transaction costs.

Using Equation (9), we can estimate efficiency every month. We can calculate the average efficiency and average turnover monthly for the last two years of each backtest. We ignore the first year, in an attempt to estimate steady-state quantities. (We start each backtest from the benchmark.)

We can also estimate the average trading costs incurred by the portfolio over those final two years of each backtest. Here we use the actual, not scaled, transaction costs.

Note that this approach optimizes expected alpha net of costs. Expected performance can fall significantly below this level when managers run portfolios suboptimally, in particular by ignoring the impact of asset levels. We will see this explicitly, as part of the example.

Expected Performance

To complete our analysis of capacity, we must compare alpha net of costs with the expected performance level, which we denote as $\bar{\alpha}$. We define capacity, A_{max} , as the asset level such that:

$$\alpha_{net} = IR_{int} \cdot \omega \cdot e(\tau) - \tau \cdot tc(A_{max}, \tau) = \bar{\alpha} \quad (11)$$

Thus we have defined capacity such that our alpha net of costs meets the performance expectations of the product.

Note that this does not mean that actual performance will always meet or exceed expectations. Our actual performance will fluctuate from year to year with a mean of α and a standard deviation of ω . At capacity, assuming a symmetrical distribution of active returns, we should

expect to beat expectations exactly half the time. Hence, as we discuss later, poor performance alone is not a good indicator of capacity problems.

We also treat the expected performance, $\bar{\alpha}$, as known and clearly agreed-upon ex ante by investors and managers. This is not always the case.

EXAMPLE

To better understand this approach, consider as an example a large-cap U.S. equity product. We assume the product operates at 5.0% active risk (a typical domestic equity mutual fund level), and that investors expect 1.4% average active returns before fees.

The product has an intrinsic information ratio of 1.2, so in the absence of any constraints or costs, this product could deliver 6% active return on average. But this is a long-only fund that faces transaction costs. We need to specify its efficiency as a function of turnover. In our experience analyzing the capacity of various products, we have used:

$$e(\tau) = e_{max} \cdot \left[1 - \text{Exp} \left\{ - \left(\frac{\tau}{\tau^*} \right)^\gamma \right\} \right] \quad (12)$$

to capture this functional dependence. This form includes a maximum efficiency, e_{max} , and shows rapid increase in efficiency for $\tau < \tau^*$ and slow increase in efficiency for $\tau > \tau^*$.

The exponent γ in Equation (12) arises because of the distribution of alpha horizons in our investment universe. If all our forecasts have the same horizon, we can model efficiency as a simpler exponential function with $\gamma = 1$. If some forecasts have shorter horizons than others, the more typical case, we observe $\gamma < 1$.

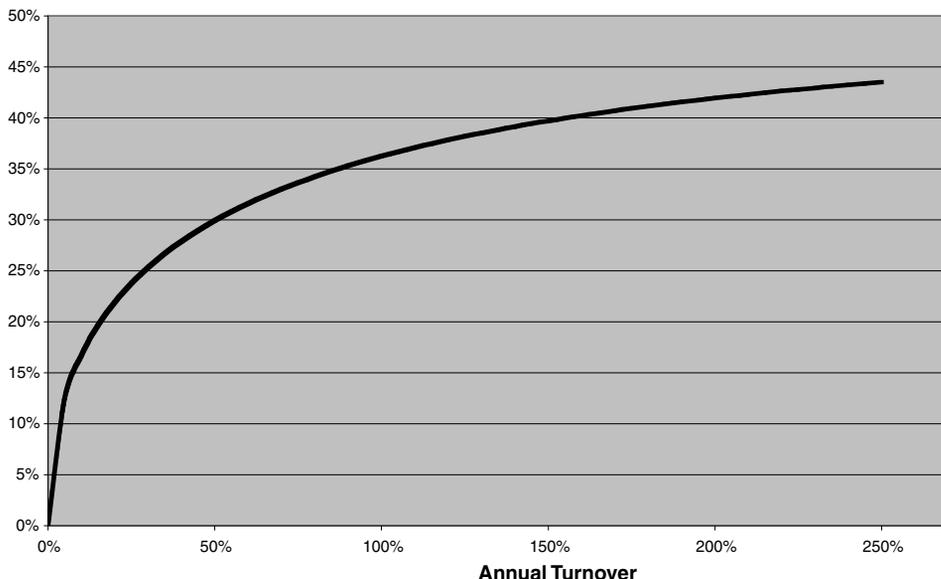
In our work on product capacity, we have often found it useful to fit functional forms to our backtest results on efficiency and average costs, and then analyze capacity according to these fitted results. This is a convenience, not a requirement.

The maximum efficiency of a long-only product declines with increasing active risk.⁵ For this example, we assume $e_{max} = 50\%$.

The parameters τ^* and γ depend on the speed of our alpha information. With very slow-moving information, low turnover can achieve most of the possible efficiency. For this example, we choose $\tau^* = 60\%$ and $\gamma = 0.5$, leading to the results graphed in Exhibit 1.

EXHIBIT 1

Efficiency



For average costs, we assume:

$$tc(A, \tau) = a + b \cdot \sqrt{A \cdot \tau} \quad (13)$$

Average trading costs include a constant term to capture average spread and commissions, plus a term that increases with dollars traded per year. The square root dependence is characteristic of many models of market impact.⁶

For this example, we assume $a = 15$ basis points, and $b = 20$ basis points per $\sqrt{\text{\$billion/year}}$. This leads to the average cost function displayed in Exhibit 2. So if we were managing \$10 billion, and had 100% turnover per year, our average trading cost would be about 80 basis points; if we turned over 200% per year, our average costs would exceed 1%.

We now have everything we need to analyze capacity. For any given asset level, we choose the turnover level that maximizes Equation (10), the alpha delivered to clients. Using Equations (12) and (13), plus our assumed parameters, Exhibit 3 displays the result.

Not surprisingly, turnover falls as assets grow. At \$10 billion in assets, the fund exhibits 62% turnover. This drops to 50% turnover as assets double to \$20 billion.

Now that we have solved for the optimal turnover for each asset level, Exhibit 4 displays alpha before costs, the costs, and the alpha net of costs, as functions of asset level. We see several behaviors we have found quite char-

acteristic of capacity studies. These include:

- Alpha net of costs decays slowly with growing assets, especially for higher asset levels.
- Costs change relatively little as assets increase. Evidently, turnover declines to almost exactly offset the increase in average costs per trade, as assets grow. This implies that managing capacity requires more than just monitoring costs.
- Performance erodes because alpha before costs erodes.

Given this behavior for alpha net of costs, what can we say about the capacity of this product? Remember that investors expect on average 1.4% alpha over time. In Exhibit 4, the alpha net of costs line crosses 1.4% at \$20 billion. This is the capacity of the product.

As a practical matter, if current assets were, say, \$2 billion, we would not use this analysis to assume we could add an additional \$18 billion without worry. Instead, we typically use the analysis to judge whether we can increase asset levels by, e.g., 20%, after which we would repeat the analysis.

Suboptimal Portfolio Management

Before analyzing the sensitivity of our results to various parameter choices, let's examine a more fundamental

EXHIBIT 2

Average Costs (A = \$10 Billion)

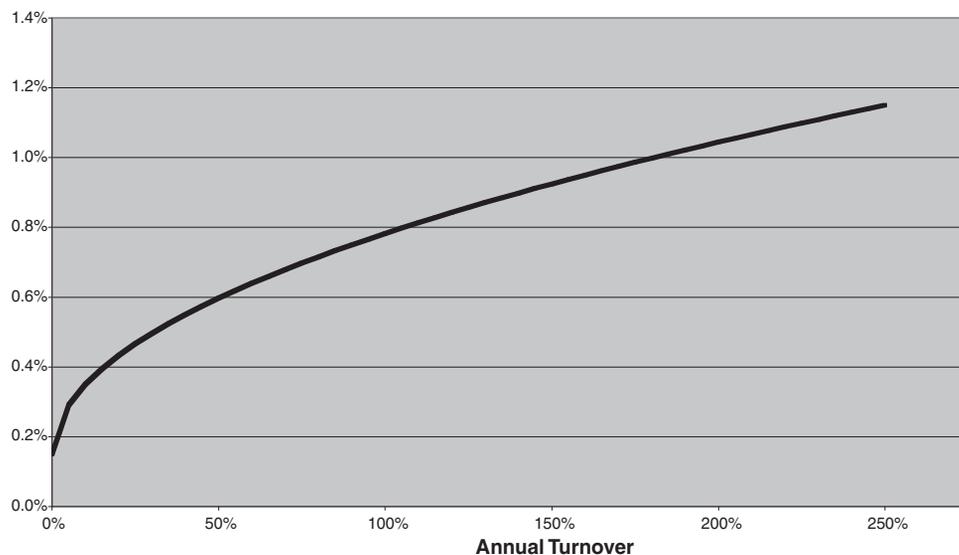
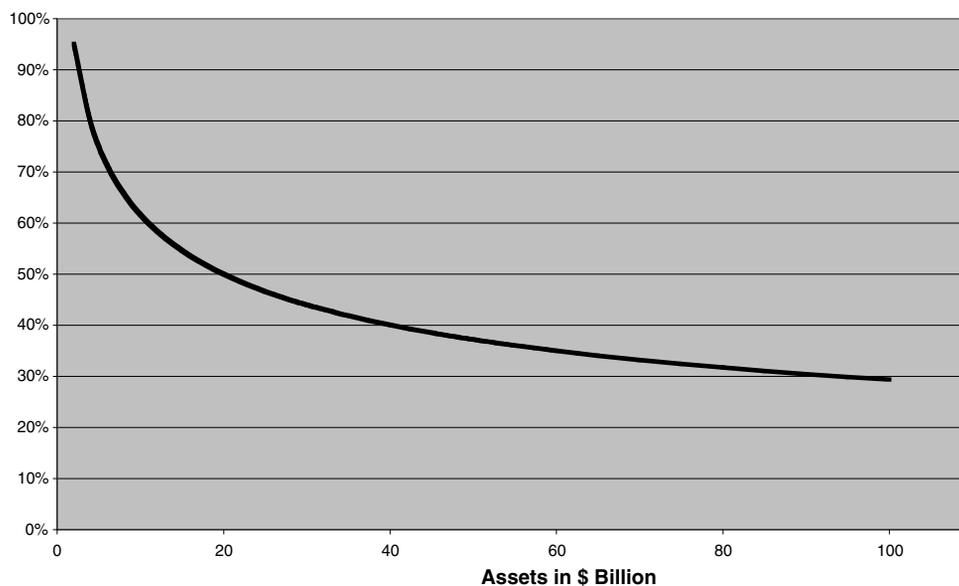


EXHIBIT 3

Optimal Turnover



issue, the impact of ignoring asset level implications in portfolio management. What would happen to alpha net of costs if our example manager keeps turnover constant as assets increase?⁷

For concreteness, we assume the manager turns over about 75% per year, the optimal turnover at \$5 billion. For lower asset levels, 75% turnover is too low. For higher asset levels, it's too high. Exhibit 5 shows the resulting alpha net

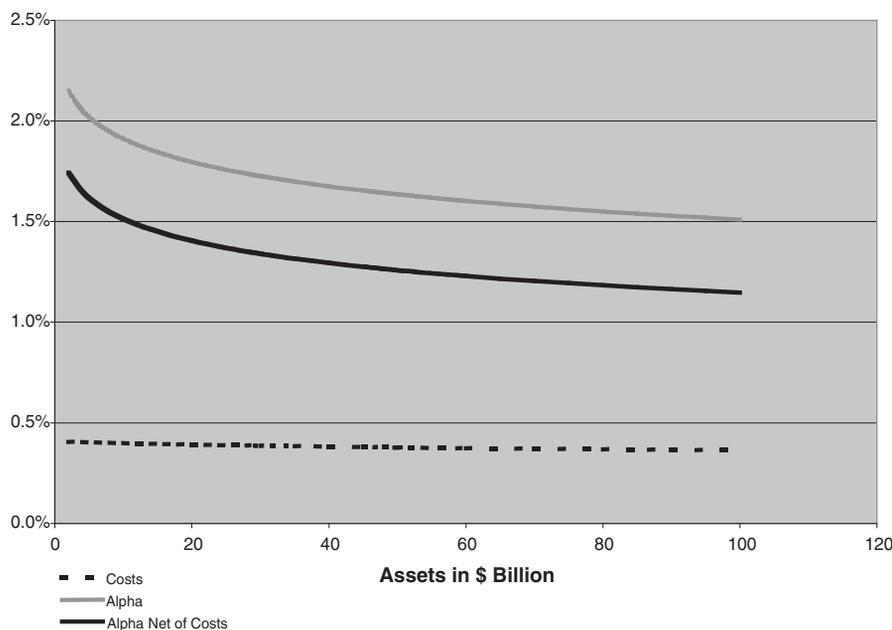
of costs compared with the optimal behavior result.

In this particular example, suboptimal portfolio management reduces capacity from \$20 billion to \$15 billion, a 25% reduction. More important, as assets exceed capacity, net alpha drops quickly, especially compared with the optimal case.

Exhibit 5 also shows the cause: steadily rising costs compared with the optimal case. So, at \$50 billion, e.g.,

EXHIBIT 4

Behavior of Alpha and Costs



optimal portfolio management can still deliver an expected 1.26% alpha net of costs. This falls to 0.99% for suboptimal portfolio management. To put it another way, at that asset level, suboptimal portfolio management leaves \$135 million per year in expected alpha on the table just through poor investment process.

Sensitivity Analysis

To provide context for understanding this capacity estimate of \$20 billion, we find it useful to examine some additional sensitivity analysis. In particular, we analyze the sensitivity to our input IR_{int} , and to the forecast costs.

Let's start with sensitivity to IR_{int} . In addition to our analysis assuming an IR_{int} of 1.2, we also analyze raising and lowering that by 0.2. Exhibit 6 displays the alpha net of costs in all three cases, plus the estimated costs in each case.

Not surprisingly, alpha net of costs increases with IR_{int} . In the absence of costs, we would expect an increase of 0.2 in IR to increase alpha by 1%, given the 5% active risk. As we can see in Exhibit 6, the alpha net of costs increases by only about 0.25%, due to the effect of constraints and transaction costs. We also operate at a higher cost level as IR_{int} increases. For an IR increase of 0.2, our costs increase by about 0.07%.

Capacity is extremely sensitive to IR_{int} . Increasing IR_{int} by 17%, to 1.4, increases capacity by a factor of five, to \$100 billion. Lowering IR_{int} by 17%, to 1.0, cuts capacity by a factor of ten, to \$2 billion

We can similarly analyze cases to estimate capacity sensitivity to transaction costs. Exhibit 7 shows the results of additional analyses, where we have simply scaled overall transaction costs, Equation (13), up or down by 20%. Effectively, we have analyzed $a = 18$ basis points and $b = 24$ basis points per $\sqrt{\text{\$billion/year}}$, and $a = 12$ basis points and $b = 16$ basis points per $\sqrt{\text{\$billion/year}}$.

According to Exhibit 7, raising costs by 20% lowers capacity by 35%, to \$13 billion. Lowering costs by 20% raises capacity by 75%, to \$35 billion.

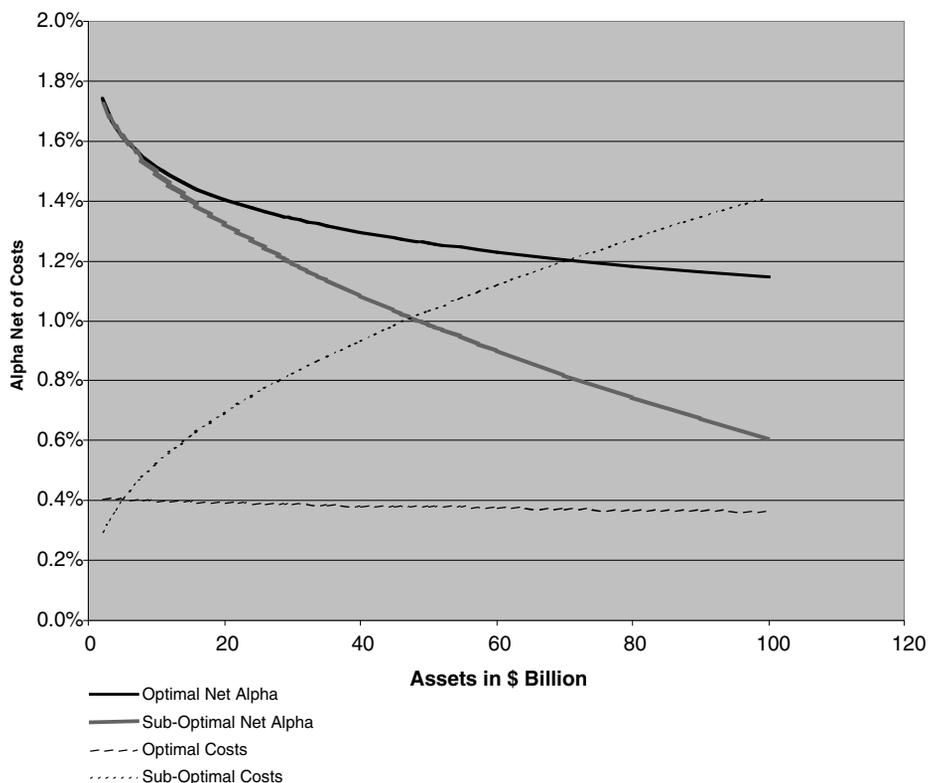
Our sensitivity analysis highlights an important point: We cannot estimate capacity with much precision. While we previously estimated capacity as \$20 billion, we should more accurately state that the capacity probably lies somewhere in a range between \$10 billion and \$50 billion.

But while capacity is extremely sensitive to IR_{int} and transaction costs, alpha net of costs is much less sensitive. For example, what if the "true" IR_{int} is 1.2, but we overestimate it to be 1.4, and set capacity to \$100 billion? We would not be able to deliver 1.4% alpha on average over time. But, in fact, we could deliver 1.15% alpha on average over time.

Exhibit 8 shows the situation. While this falls short

EXHIBIT 5

Impact of Suboptimal Portfolio Management



of expectations by 0.25% per year, that is a surprisingly small shortfall for overestimating capacity by a factor of five. For example, it would increase our probability of delivering a negative active return over a one-year period from 39% to 41%, hardly noticeable over any period of reasonable interest.

Exhibits 9 and 10 summarize this case.

And, of course, the extreme sensitivity of capacity relates directly to the relative insensitivity of alpha net of costs. Because alpha changes so slowly with assets, moving the curve up or down small amounts will move capacity by large amounts. Fortunately, it appears that errors in capacity do not lead to substantial errors in deliverable alpha over time.

We should note that our analysis of sensitivity does depend on our choice of expected performance. If expected performance is, e.g., 1.65% (a target set where alpha net of costs declines more steeply with assets), we would observe somewhat lower sensitivity (and a correspondingly more precise capacity estimate).

This analysis of the sensitivity of estimated capacity and alpha net of costs tells us two things. First, we cannot estimate capacity very accurately. Second, inaccurate estimates do not lead to dramatic performance problems.

SHORTCOMINGS OF THE APPROACH

We have presented a reasoned and clear approach for analyzing capacity, and applied it to a particular example. Even so, this analysis falls short in several areas, which we can categorize by impact on our fundamental result, Equation (10).

First, our models of trading costs break down as trading volumes approach significant fractions of average daily volume. At these high levels, some trades are not completed, regardless of price. More generally, this approach requires analysis of trading volumes far beyond one's typical experience. These shortcomings will lead us to underestimate costs in general, and hence overestimate capacity, according to Equation (10).

In analyzing long-short capacity, we cannot accurately anticipate problems with locating stocks to borrow as assets increase. We also cannot easily account for other market constraints such as poison pills, which become important at high asset levels. These important market and regulatory issues will reduce our efficiency. And perhaps more important, the analysis ignores the impact of competitors following similar investment strategies. That can

EXHIBIT 6 Sensitivity to IR

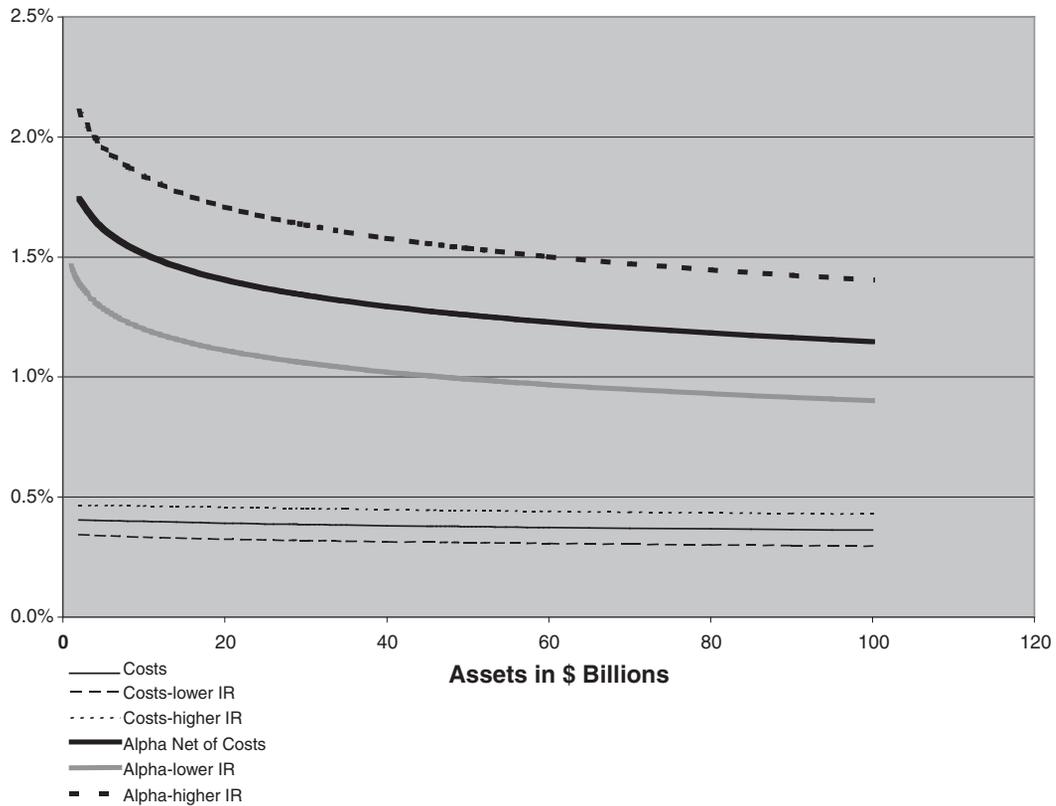


EXHIBIT 7 Sensitivity to Trading Costs

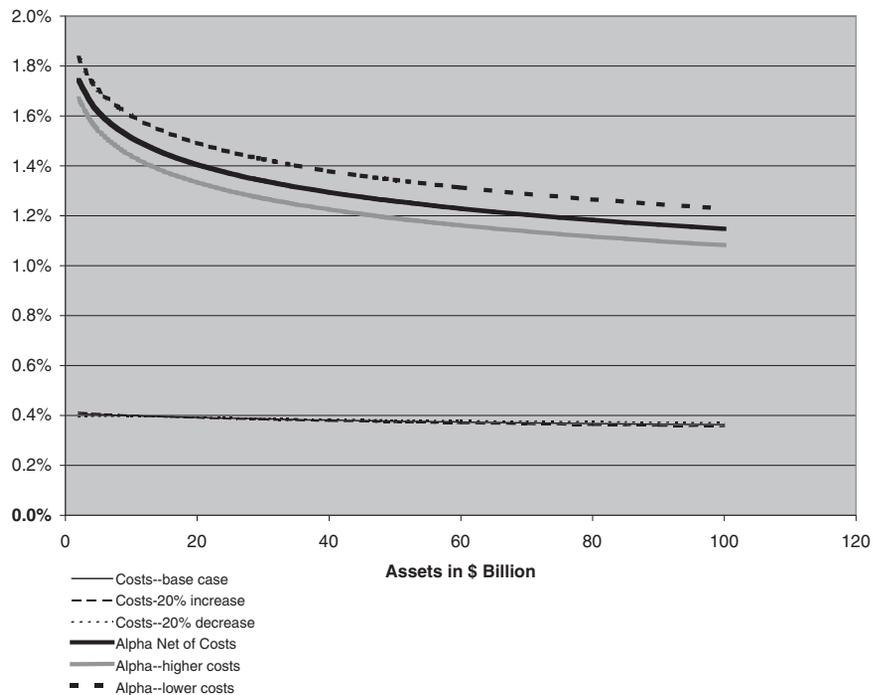


EXHIBIT 8

Active Return Distribution

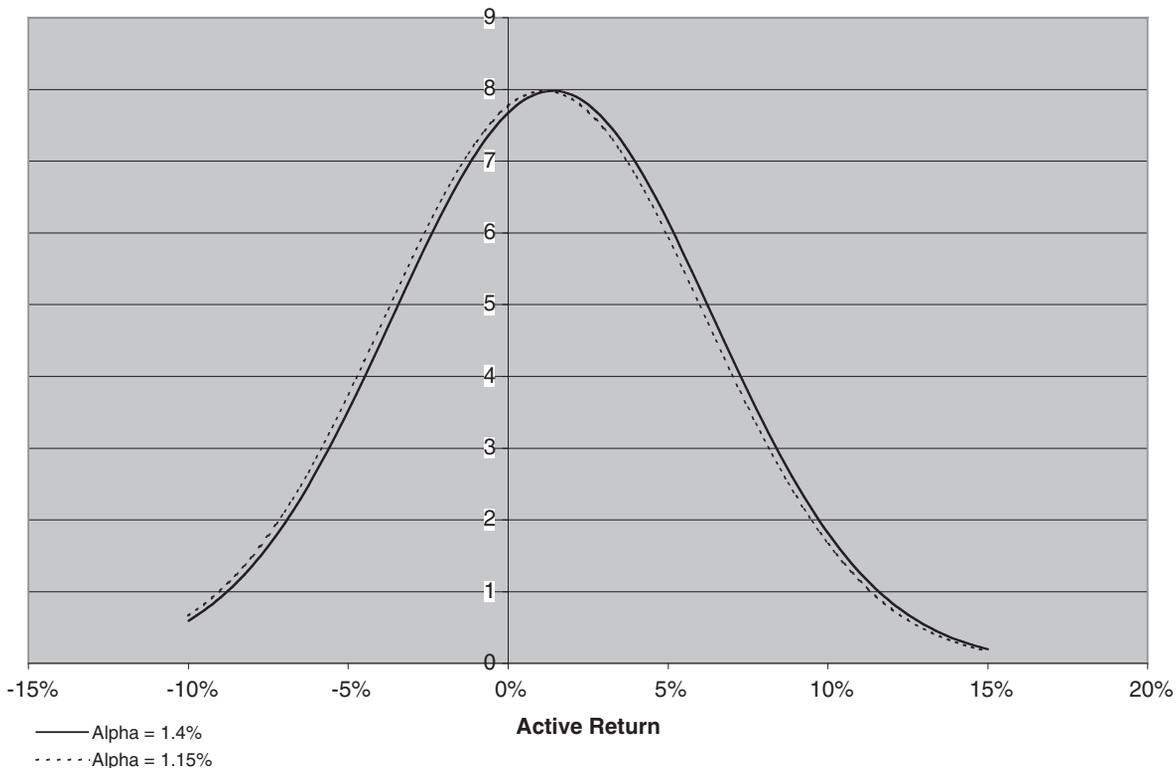


EXHIBIT 9

Capacity Sensitivity

		Capacity in \$ Billion		
		IR = 1.0	IR = 1.2	IR = 1.4
cost scaling	1.2	1	13	68
	1.0	2	20	100
	0.8	4	35	170

EXHIBIT 10

Net Alpha Sensitivity

		Alpha, A = \$30 Billion		
		IR = 1.0	IR = 1.2	IR = 1.4
cost scaling	1.2	1.00%	1.27%	1.55%
	1.0	1.06%	1.34%	1.63%
	0.8	1.13%	1.43%	1.73%

reduce our intrinsic information ratio by arbitraging away some of our ideas.

This methodology also falls short in assuming we can always reach the target risk level. At very high asset levels, we may find this impossible. The trading costs are too high to build up and maintain the active positions consistent with target risk.

Reducing our efficiency, intrinsic information ratio, or risk level will lower the first term on the right-hand side of Equation (10), implying that we overestimate capacity if we ignore these effects.

One way to see an overall shortcoming here is by real-

izing that our slow decay of alpha net of costs implies that dollars of alpha (i.e., $A \cdot \alpha$) keep increasing with asset level. If this is true, the Perold and Salomon [1991] methodology would thus imply infinite capacity. At extreme asset levels, however, we must reach a maximum of dollars of alpha. This maximum will depend on the number of available stocks, and on institutional and trading constraints.

Our avoidance of these real-world issues can lead to overestimates of capacity. In fact, these real-world issues ultimately limit capacity, given that this framework implies capacity is typically very high, assuming optimal portfolio management. To mitigate this problem, we typically

use capacity analysis to investigate only whether we can increase asset levels by, e.g., 20%. This helps avoid extrapolating far beyond our experience. We then manage asset growth, and regularly analyze where we are relative to available capacity as assets increase.

The analysis also assumes optimal portfolio construction, given alphas, risks, costs, and constraints. As we have seen, poor portfolio construction can artificially limit capacity. Consider, for example, the technique of buying one's top 50 stocks and equal-weighting them. Such an approach ignores any impact of asset levels. The approach will have significantly lower capacity than a product with similar alpha views, but the ability to increase the number of stocks if the trade-off between lower alpha before costs (in going beyond the top 50 stocks) and lower costs (reducing impact by trading across more stocks) makes that worthwhile. This will also lead to overestimates of capacity for suboptimally managed products.

MONITORING CAPACITY

We have mostly described how managers should analyze their own products. Now we describe how to monitor capacity, including using some aspects accessible to sponsors and consultants.

Monitoring returns is not the same as monitoring capacity. Asset levels significantly above capacity can erode alpha net of costs, but shouldn't actually generate negative returns, except through suboptimal portfolio management. The Magellan Fund's negative performance cannot by itself tell us if it was out of capacity in August 1997.

We have seen that alpha net of costs typically decays quite slowly with increasing assets, and that in optimal approaches trading costs remain quite constant over a wide range of asset levels. Hence, monitoring realized transaction costs at the portfolio level will not flag the problem. (It will, however, identify managers who invest suboptimally.)

The most recognizable characteristic of this issue is the decay in alpha before costs as asset levels increase—so managers should monitor this quantity.

We can observe another characteristic by examining the product's risk budget. The product aims for a target active risk level. At low asset levels, it distributes that risk mainly based on alpha and risk considerations. As assets and costs increase, the product uses more of its risk budget in the most liquid stocks. This can cause problems if alphas are higher for less liquid stocks—a not atypical situation.

To monitor capacity, we should also monitor the risk allocation within the product. Sponsors and consultants

should monitor, in particular, the fraction of active risk allocated to the most liquid stocks in the investment universe.

Finally, we have noted that costs remain fairly constant as assets increase. This occurs because turnover declines, and the fund spreads out trades over more stocks, as assets increase. Sponsors and consultants should expect to see these behaviors in optimally run portfolios as assets grow. And, they should grow concerned if they do not observe lower turnover and an increasing number of stock positions as product assets grow.

INCREASING CAPACITY

This analysis also points to at least two general approaches to increasing capacity. Capacity is very sensitive to intrinsic information ratios and transaction costs. Small improvements in those quantities will have bigger impacts on capacity. As Grinold [1989] shows, we can improve the intrinsic information ratio through increased skill (information coefficient) or increased breadth (number of independent bets per year). Research on new and better investment ideas can increase capacity.

Sensitivity to transaction costs motivates research and development of trading strategies and approaches designed to lower costs.

CONCLUSIONS

Capacity is an important concern for managers, sponsors, and consultants. We have prescribed a framework for analyzing capacity, and have observed several surprising behaviors:

- Alpha net of costs decays slowly with growing asset levels.
- Capacity is quite sensitive to the assumed intrinsic information ratio and costs.
- For this reason, we cannot precisely estimate capacity. Inaccuracies in such estimates have limited impact on alpha net of costs.
- Suboptimal portfolio management substantially lowers capacity.
- We can monitor capacity, although negative returns do not signify capacity problems, and increasing costs usually signify suboptimal portfolio management.
- Managers can increase capacity by increasing their intrinsic information ratio or lowering their costs.

ENDNOTES

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¹Private communication with William Muysken, Global Head of Research, Mercer Investment Consulting.

²We will ignore the distinction between active and residual returns in this analysis, or, equivalently, assume that the portfolio has $\beta = 1$.

³This concept appears in Grinold and Kahn [2000a], pp. 433-436; Grinold and Kahn [2000b]; and as the transfer coefficient in Clarke, de Silva, and Thorley [2002].

⁴For a derivation of this relationship, see Grinold and Kahn [2000a], pp. 135-137.

⁵Grinold and Kahn [2000b]. Higher levels of active risk require higher overweights and underweights, which run into the long-only constraint more often.

⁶See, for example, Grinold and Kahn [2000a], pp. 450-454. The "BARRA Market Impact Model Handbook" [1997], p. 85, also provides quantitative evidence for this dependence.

⁷This describes not only many investment managers, but also all academic studies in this area.

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