THE JOURNAL OF FINANCE • VOL LIII, NO. 2 • APRIL 1998

On Stable Factor Structures in the Pricing of Risk: Do Time-Varying Betas Help or Hurt?

ERIC GHYSELS*

ABSTRACT

There is now considerable evidence suggesting that estimated betas of unconditional capital asset pricing models (CAPMs) exhibit statistically significant time variation. Therefore, many have advocated the use of conditional CAPMs. If we succeed in capturing the dynamics of beta risk, we are sure to outperform constant beta models. However, if the beta risk is inherently misspecified, there is a real possibility that we commit serious pricing errors, potentially larger than with a constant traditional beta model. In this paper we show that this is indeed the case, namely that pricing errors with constant traditional beta models are smaller than with conditional CAPMs.

LINEAR FACTOR MODELS such as the unconditional capital asset pricing model (CAPM) and the arbitrage pricing theory (APT) have been the cornerstone of theoretical and empirical finance for decades now. Supported by seminal papers, like those of Sharpe (1964), Lintner (1965), Merton (1973), and Ross (1976), they are the most widely used tools to value the return on risky assets. Although the theory maintains a linear and stable relationship between risk factors and returns, there is now considerable empirical evidence documenting time variation in market betas and other factor payoffs. This is perhaps not so surprising because the theoretical underpinnings of the unconditional arbitrage-pricing theory reveal that time-invariant linear factor structures are only obtained when one imposes strong assumptions on the underlying probability distributions and investors' attitudes toward risk.[1]

In practice, many portfolio managers constantly update and reestimate factor returns, and indeed Harvey (1989), Ferson and Harvey (1991, 1993),

*Department of Economics, Pennsylvania State University, and CIRANO. We would like to thank Wayne Ferson and Campbell Harvey for their invaluable help in providing their data. We are also most grateful to two referees and to the editor, René Stulz, for their very helpful comments. Benoit Durocher provided excellent research assistance. We would also like to thank Cliff Ball, Ravi Bansal, Wayne Ferson, René Garcia, Eric Jacquier, Eric Renault, Pierre Séquier, S. Viswanathan, and participants at the CREST–Compagnie Bancaire Conference, the 1995 Western Finance Association and American Finance Association Meetings, the Brazilian Econometric Society Meetings, HEC Paris, and Penn State for their comments.

[1] Several general equilibrium developments of the unconditional CAPM and APT have been advanced; for example, see Huberman (1982), Chamberlain and Rothschild (1983), Ingersoll (1984), Connor (1984), Connor and Korajczyk (1989), among others.

and Ferson and Korajczyk (1995) find that estimated betas exhibit statistically significant time variation. There appears to be a consensus now about the failure of the static CAPM; for example, see the widely cited work of Fama and French (1992) and the discussion it generated. Some advocate that the static CAPM should be replaced by some form of time-varying beta conditional CAPM (see, e.g., Jagannathan and Wang (1996)). If we succeed in capturing the dynamics of beta risk, we are sure to outperform constant beta or so-called unconditional CAPM and APT models. But what if we do not succeed in correctly specifying time-varying betas? Then, if beta risk is inherently misspecified, there is a real possibility that we commit serious pricing errors that potentially could be bigger than with a constant beta model. In this paper, we show that this is not purely an abstract speculation or remote possibility. Indeed, we show that in many cases the pricing errors with constant beta models are *smaller* than those with time-varying beta models. The misspecification of the latter appears to be serious enough that time-varying beta models do not help but actually hurt.

The models we consider are several conditional CAPM and APT dynamic asset pricing models. Ferson (1985), Ferson and Harvey (1991, 1993), Harvey (1991), Ferson and Korajczyk (1995), and Dumas and Solnik (1995), among others, have applied these models to price international equities, bonds, and size-sorted and industry-based portfolios, as well as forward currency contracts.

How do we find out that these time-varying beta models are misspecified? One way, perhaps somewhat ironically, it is to investigate the exact same issue that motivated the replacement of the unconditional CAPM and APT by conditional models. Indeed, misspecification in beta risk dynamics is almost always revealed by nonconstancy of the beta risk model parameters (just as misspecification of the CAPM is revealed by time variation in the betas). Hence, we test whether there are structural shifts in the parameters of conditional CAPM and APT models and find overwhelming evidence for structural breaks. This means that conditional CAPM and APT models previously presented in the literature are misspecified.

In Section I we discuss the impact of misspecification on pricing errors explaining intuitively ways to assess misspecification through testing for parameter constancy. Section II presents a brief review of the conditional CAPM and APT and the nonlinear APT models of Bansal and Viswanathan (1993) and Bansal, Hsieh, and Viswanathan (1993). Although nonlinear APT models do not provide explicit pricing formulas, it is of interest to examine their constancy and compare them with the other models. Using a uniform data set across all models, we report in Section III a comprehensive empirical study with monthly NYSE stock returns. The results reveal some serious specification problems. On that basis we proceed with the analysis of pricing errors. In Section IV we report results showing a strong dominance of the traditional CAPM in predicting returns for size-sorted and industry-based portfolios. Section V concludes the paper.

I. Predicting Asset Returns and Specification Errors

Let us concentrate on a very simplified version of the conditional CAPM to set the stage for our discussion:

$$E[r_{it+1}|Z_t] = \beta_{it} E[r_{Mt+1}|Z_t], \tag{1}$$

where β_{it} is the parameterized time-varying market beta and Z_t is a set of instruments. The excess return from t to $t+1$ on the market portfolio is measured by r_{Mt+1}, and r_{it+1} is the excess return on any asset or portfolio of assets i. Once we admit that beta varies through time we must specify laws of motion for β_{it}. A conditional CAPM does precisely that; namely, with a single instrument it implies the following:

$$\beta_{it} = \frac{E[(r_{Mt+1} - \delta_M Z_t)(r_{it+1} - \delta_i Z_t)|Z_t]}{E[(r_{Mt+1} - \delta_M Z_t)^2|Z_t]}. \tag{2}$$

From equation (2) we note that two fixed parameters, namely δ_M and δ_i, together with Z_t, r_M, and r_i, determine the time variation in β_{it}. The two parameters are obtained via the projection equations

$$E[r_{jt+1}|Z_t] = \delta_j Z_t \qquad j = i, M. \tag{3}$$

The question we are interested in is whether this particular (or any other) characterization of β_{it} is adequate and does not yield a systematic mispricing of risk factors. Combining equations (1) and (3), we can write the asset pricing equation as follows:

$$r_{it+1} = \beta_{it}\delta_M Z_t + u_{it+1}, \tag{4}$$

where $E(u_{it+1}Z_t) = 0$. If the restrictions of the conditional CAPM do not hold because beta risk is inherently misspecified, we obtain as a generic alternative

$$r_{it+1} = \tilde{\beta}_{it}\tilde{\delta}_{Mt} Z_t + \tilde{u}_{it+1}, \tag{5}$$

with $E(\tilde{u}_{it+1}Z_t) = 0$ and $\tilde{\beta}_{it} \neq \beta_{it}$ obtained from equation (2) replacing δ_M by $\tilde{\delta}_{Mt}$ and δ_i by $\tilde{\delta}_{it}$. It should be noted that the generic alternative in equation (5) emphasizes the fact that the specification of β_{it} is erroneous. Other sources of misspecification, such as omitted factor risk, are, at least for the moment, not considered. No specific laws for $\tilde{\delta}_{Mt}$ or $\tilde{\delta}_{it}$ (and hence $\tilde{\beta}_{it}$) are used at this point. To assess the consequences of misspecification we will compare two different scenarios. The first scenario is one in which *both* the traditional fixed beta CAPM and the conditional CAPM are misspecified. Under the second scenario *only* the unconditional CAPM is erroneous. We study pricing errors under both scenarios, assuming that we can ignore estimation uncertainty. When the conditional CAPM is correctly specified (i.e., the second scenario) it should be better at predicting asset returns. Let us formalize this intuition before moving to the first scenario where both models are misspecified. To examine the magnitude of pricing errors we consider the mean squared error (MSE). For the conditional CAPM we have

$$MSE_{2C}^{i} \equiv E[r_{it+1} - \beta_{it}\delta_M Z_t]^2 = \text{Var}(u_{it+1}), \tag{6}$$

where MSE_{jk}^{i} is the mean squared error for asset i under scenario $j = 1, 2$, and model $k = C$ for conditional CAPM and $k = U$ for the unconditional one. To determine MSE_{2U}^{i} let us write equation (2) as $\beta_{it} \equiv \beta_i + \beta(Z_t;(\delta_i,\delta_M))$, where β_i is the (fixed unconditional) beta for asset i.[2] Then we obtain

$$\begin{aligned} MSE_{2U}^{i} &\equiv E[(\beta_i + \beta(Z_t;(\delta_i,\delta_M)))\delta_M Z_t + u_{it+1} - \beta_i \delta_M Z_t]^2 \\ &= \text{Var}(u_{it+1}) + \delta_M^2 E[\beta(Z_t;(\delta_i,\delta_M))Z_t]^2 + 2\delta_M E\beta(Z_t;(\delta_i,\delta_M))Z_t u_{it+1}. \end{aligned} \tag{7}$$

The last term on the right-hand side of equation (7) is zero since u_{it+1} is orthogonal to any function of Z_t. Hence, we have that MSE_{2U}^{i} is larger than MSE_{2C}^{i} by a factor of $\delta_M^2 E[\beta(Z_t;(\delta_i,\delta_M))Z_t]^2 > 0$. Clearly, as there is evidence of time-varying beta we have an obvious interest in adequately modeling time-varying betas and exploiting them for pricing securities. But what if the time-varying betas are prone to specification error? This is precisely the first scenario. Under equation (5) we can compute MSE_{1C}^{i} and MSE_{1U}^{i}. Some algebra yields the following:

$$\begin{aligned} MSE_{1U}^{i} - MSE_{1C}^{i} = \delta_M^2 E[\beta(Z_t;(\delta_M,\delta_i))Z_t]^2 \\ - E[(\tilde{\beta}_{it}\tilde{\delta}_{Mt} - \beta_i\delta_M)\beta(Z_t;(\delta_M,\delta_i))\delta_M Z_t^2]. \end{aligned} \tag{8}$$

The second term on the right-hand side of equation (8) can be negative, or positive and larger than the first term. Hence, depending on its magnitude and sign, we may have $MSE_{1U}^{i} > MSE_{1c}^{i}$, which means the conditional CAPM yields the smallest pricing error; but we can also have the reverse, $MSE_{1U}^{i} < MSE_{1C}^{i}$. A priori we do not really know, as it all depends on how severely the conditional CAPM is misspecified relative to the unconditional one. In the next section we present the conditional APT and CAPM we will consider to investigate this question. In Section III we explain how to test that the dynamics of betas are misspecified.

[2] The functional $\beta(\cdot)$ is introduced to highlight the fact that in equation (2) beta risk is implicitly a function of Z_t and parameterized by δ_M and δ_i.

II. A Review of the Conditional Asset Pricing Models

We follow Bansal and Viswanathan (1993) closely and start from the optimal portfolio allocation conditions of discrete time capital asset pricing models.[3] In an economy with N assets we obtain the following first-order conditions:

$$E[MRS(t,t+1)x_{it+1}|\Omega_t] = \Pi(x_{it+1}) \qquad \text{for } i = 1,\ldots,N, \tag{9}$$

where x_{it+1} is the one-period payoff of the ith asset at time $t+1$ that has time t price $\Pi(x_{it+1})$ and where $MRS(t,t+1)$ is the representative agent's marginal rate of substitution between t and $t+1$ consumption. The expectation in equation (9) is conditional on the information set Ω_t. Equation (9) also holds when we replace $MRS(t,t+1)$ by its projection on the space of all one-period payoffs. Let us denote this projection as P^*_{t+1}. Hansen and Jagannathan (1991) show that this projection can be expressed as a linear combination of the N asset one-period payoffs represented by the vector $x_{t+1} = [x_{it+1}]_{i=1}^N$:

$$P^*_{t+1} = \sum_{j=1}^{N} \alpha_{jt} x_{jt+1}, \tag{10}$$

where the weights $\alpha_t = [\alpha_{jt}]_{j=1}^N$ satisfy

$$\alpha_t = [E[x'_{t+1}x_{t+1}|\Omega_t]]^{-1}\Pi(x_{t+1}). \tag{11}$$

Equations (10) and (11) represent a fundamental relationship in characterizing the pricing of assets but they do not yet comprise a "workable" model, which involves as many factors as there are assets, namely N basis portfolios. To make the model workable we need to reduce the set of factors, yet equations (10) and (11) tell us that this is unlikely to be attainable with a simple fixed linear relationship. This observation yielded the nonlinear APT of Bansal and Viswanathan (1993) and Bansal et al. (1993) and the conditional CAPM and APT of Ferson (1985) and Ferson and Harvey (1991, 1995), among others. We shall begin by briefly presenting the former and then continue with the latter class of models. For the nonlinear APT we use equation (10) and replace the marginal rate of substitution by its projection onto Ω_{t+1}, yielding

$$E[E[MRS(t,t+1)|\Omega_{t+1}]x_{it+1}|\Omega_t] = \Pi(x_{it+1}). \tag{12}$$

[3] See Lucas (1978), Breeden (1979), Stulz (1981), Huang (1987), Duffie and Zame (1989) among others.

Then, instead of using the projection onto the entire information we consider a vector P^b_{t+1} of well-diversified basis variables such that

$$E[MRS(t,t+1)|\Omega_{t+1}] = E[MRS(t,t+1)|P^b_{t+1}] = G(P^b_{t+1}), \tag{13}$$

with $G(\cdot)$ a well-behaved function chosen among a class of flexible functional forms. Using the fact that $\Pi(x_i(t,t+1)) \in \Omega_t$ and normalizing equation (12) yields the following set of moment conditions:

$$E[(G(P^b_{t+1})x_{it+1} - 1)Z_t] = 0, \tag{14}$$

where Z_t is a set of instruments picked among the elements of Ω_t. Equation (14) forms the basis of a GMM estimation procedure for the parameters describing the pricing kernel $G(\cdot)$. The set of Z_t instruments actually used in our empirical work will be described later since it coincides with those used in the conditional APT model. The elements entering P^b_{t+1} are the same as those used by Bansal et al. (1993) in their one-factor model, namely

$$P^b_{t+1} = (1 + r_{Mt+1}, 1 + r_{ft+1}), \tag{15}$$

where r_{Mt+1} is the nominal return on the market and r_{ft+1} is the nominal yield to maturity on the Treasury bill next period. What remains to be specified is a functional form for $G(\cdot)$. As the exact specification of the nonlinear pricing kernel is unknown, Bansal et al. (1993) suggest approximating it with a polynomial series expansion,[4] namely

$$G(P^b_{t+1}) = \beta_0 + \beta_{1t} r_{ft+1} + \sum_{j=1,2,5} \beta_{jM}[r_{Mt+1}]^j. \tag{16}$$

With regard to the asset x_i appearing in equation (14), we shall consider a set of size-sorted portfolios and industry-based classified portfolios that will also be used in the conditional APT. The details are discussed in Section III.C.

We turn now our attention to the conditional CAPM considered by Harvey (1991) to study the pricing of international assets and used by Ferson and Korajczyk (1995) to study predictable returns and risk in the United States. Again, one can start from the observation that equations (10) and (11) do not directly yield a workable model, but instead of considering a nonlinear pricing kernel, Harvey proposes to study expected returns for stock markets from a set of countries via their *conditional* beta with the return on a world market portfolio. Harvey (1991) shows that one obtains a set of moment conditions suitable for Generalized Method of Moments (GMM; see Hansen (1982)) estimation of $\delta = [\delta_i]_{i=1}^N$ *and* δ_M as follows:

[4] As Bansal and Viswanathan (1993) explain, using the fifth order rather than the third was partly motivated by the need to reduce collinearity between the various powers of the expansion.

$$E\begin{pmatrix} (r_{t+1} - Z_t\delta)' \\ (r_{Mt+1} - Z_t\delta_M)' \\ (u_{Mt+1}^2 Z_t\delta - u_{Mt+1}u_{t+1}Z_t\delta_M)' \end{pmatrix}' \otimes Z_t' = 0, \tag{17}$$

where $r_{t+1} = [r_{it+1}]_{i=1}^N$, $u_t = r_t - Z_{t-1}\delta$, and $u_{Mt} = r_{Mt} - Z_{t-1}\delta_M$.

We also investigate an alternative specification for the conditional CAPM, one suggested by Ferson and Harvey (1993), which models time-varying betas as $\beta_{it} = Z_t\beta_{ic}$ to avoid the wieldy specification of the ratio of conditional covariance and conditional variance. It yields the moment conditions

$$E\begin{pmatrix} (r_{t+1} - Z_t\delta)' \\ (r_{Mt+1} - Z_t\delta_M)' \\ (Z_t\delta - Z_t\beta_c Z_t\delta_M)' \end{pmatrix}' \otimes Z_t' = 0, \tag{18}$$

which, in contrast to equation (17), is a just-identified set of moment conditions.

In a recent paper Ferson and Korajczyk (1995) undertake a very thorough empirical investigation of risk and return for the United States using a multifactor conditional APT. The setup is very similar to that described in equation (17) except that the moment conditions are a bit more elaborate because of the presence of a multitude of factors. For the multifactor conditional APT, Ferson and Korajczyk define the following set of moment conditions:

$$E\begin{bmatrix} r_{it+1} - Z_t'\delta_i \\ (F_{t+1}' - Z_t'\gamma_i)' \\ (F_{t+1}' - Z_t'\gamma_i)(F_{t+1}' - Z_t'\gamma_i)'\beta_i - F_{t+1}(r_{it+1} - Z_t'\delta_i) \end{bmatrix} \otimes Z_t' = 0, \tag{19}$$

where F_t is a $K \times 1$ vector of factor-mimicking portfolios, β_i is a $K \times 1$ vector of the betas for asset i, and Z_t is a $(L + 1)$ vector of instruments. In contrast to the nonlinear APT and conditional CAPM, the model defined in equation (19) has parameters that play a very different role. It also makes hypothesis testing more interesting. Indeed, this more elaborate model has the advantage of separating projection equations and asset pricing moment conditions involving conditional betas. In equation (19) the third set of moment conditions does not involve any new parameters, whereas in equation (19), and equation (18) as well, the third set involves explicitly parameterized betas.

III. Empirical Results on Parameter Stability

We turn our attention now to the empirical evidence regarding the structural invariance of the three dynamic asset pricing models described in the previous section. We devote a subsection to each of these empirical models. The first subsection briefly discusses tests for structural shifts. Then we study the conditional CAPM of Harvey (1991). The next subsection covers the multifactor model of Ferson and Korajczyk (1995). We conclude with the nonlinear APT.

To facilitate comparisons, we apply the three empirical asset pricing models to the same data. Because the study by Ferson and Korajczyk (1995) has a very wide and comprehensive set of U.S. equity return series, we use their data set.[5] It consists of size-sorted returns for stocks appearing on the CRSP tapes as well as those same asset returns classified by industry. All portfolios are value-weighted. A total of 12 industry-sorted portfolios will be considered in addition to the 10 size category portfolios. We follow step by step the specification of variables and instruments described by Ferson and Korajczyk.[6]

A. *Testing for Structural Breaks*

Anyone familiar with the empirical evidence may find it surprising that there is a need to test for structural change because conditional CAPM and APT models are typically well supported by the data. To clarify this we have to elaborate on the fact that testing for structural breaks is far more stringent than the usual overidentifying restrictions tests, often called J-statistics, commonly used to diagnose the fit of an asset pricing model like the conditional CAPM. Because such models are estimated via GMM, let us proceed by specifying the moment conditions of such a model. Taking a simple example as in equation (17) with a single asset i yields

$$E\begin{pmatrix} r_{it+1} - \tilde{\delta}_{it} Z_t \\ r_{Mt+1} - \tilde{\delta}_{Mt} Z_t \\ \tilde{\delta}_{it} Z_t [(r_{Mt+1} - \tilde{\delta}_{Mt} Z_t)^2] - (r_{Mt+1} - \tilde{\delta}_{Mt} Z_t)(r_{it+1} - \tilde{\delta}_{it} Z_t)\tilde{\delta}_{Mt} Z_t \end{pmatrix} Z_t = 0. \tag{20}$$

The formulation in equation (20) represents the set of moment conditions involved in the GMM estimation procedure but does not impose the null hypothesis of constant parameters. For the moment the variation in the parameters is left unspecified; ultimately they will represent structural breaks as will be discussed shortly. The estimation of the conditional CAPM imposing fixed parameters δ_M and δ_i with the data generated by equation (20) will yield GMM parameter estimates $\bar{\delta}_M$ and $\bar{\delta}_i$, which are some sort of sample

[5] Hence, the results we will report for Harvey's conditional CAPM will not exactly replicate his original study of international excess stock returns. Yet empirical results we obtain with Harvey's original data series are completely in line with those we are about to discuss. These results are not reported but are available upon request. Similarly, the results reported for the nonlinear APT model do not correspond to the original empirical work, but the conclusions we draw from our investigation are nevertheless representative.

[6] We actually directly use the data set they constructed. This also applies to the factor-mimicking portfolios that are discussed in Section III.C.

averages of the underlying $\tilde{\delta}_{Mt}$ and $\tilde{\delta}_{it}$. Ghysels and Hall (1990b) show formally that overidentifying restrictions tests based on the moment conditions such as those in equation (20) but evaluated at fixed parameter estimates $\bar{\delta}_M$ and $\bar{\delta}_i$ have a tendency *not* to reject the model. This problem is not just a theoretical curiosity. Indeed, we will provide numerous examples where this situation occurs in empirical asset pricing models. Hence, the usual diagnostic tests to judge the validity of a model are not adequate to detect systematic mispricing of asset returns because of erroneous beta dynamics.

Testing for structural invariance of the model amounts to verifying whether the following hypothesis holds

$$H_0\colon \begin{cases} \tilde{\delta}_{Mt} = \delta_M & \forall t = 1,\ldots,T \\ \tilde{\delta}_{it} = \delta_i & \forall t = 1,\ldots,T. \end{cases} \tag{21}$$

A great variety of tests for structural change for models estimated by GMM exist.[7] The majority of tests assume as an alternative that at some point in the sample there is a single structural break; for instance,

$$\tilde{\delta}_{jt} = \begin{cases} \delta_{j1} & t = 1,\ldots,\pi T \\ \delta_{j2} & t = \pi T + 1,\ldots,T \end{cases} \qquad j = M, i, \tag{22}$$

where π determines the fraction of the sample before and after the assumed break point.[8] If the break point πT were known, our task would be relatively easy to perform. For example, calculating δ_{j1} and δ_{j2} and comparing both estimates to see whether they are significantly different would be one way to proceed (often referred to as a Chow test). Unfortunately, in the present context we don't really want to assume π known. In recent years several procedures have been advanced to test the null hypothesis (21) against an alternative such as equation (22) with unknown break point π. In the remainder of the section we will explain what these procedures amount to (the technical details appear in the references provided in footnote 7).

We use the Sup LM, or supremum LM, test proposed by Andrews (1993). One computes the supremum of all LM tests, or score tests, over all possible break points πT. Andrews suggests this type of test and tabulates its distribution under the null hypothesis appearing in equation (21). The Sup LM test has the great advantage that it only uses the parameter estimates $\bar{\delta}_M$

[7] Relevant references include Andrews (1993), Andrews and Ploberger (1994), Ghysels, Guay, and Hall (1998).

[8] It is worth noting that in equation (22) all parameters are tested jointly for stability. In several circumstances, however, the parameters involved play different roles; therefore, depending on which ones are subject to breaks, a different interpretation should be given. For instance, in the multifactor models appearing in equation (19), one has a set of parameters δ_i and γ_i that arise from purely ancillary statistical assumptions regarding projection equations rather than with an economic interpretation. To emphasize this distinction we will often conduct tests involving only a subset of the parameter vector.

and $\bar{\delta}_i$ obtained from the full sample. This saves an enormous amount of computer time by avoiding all the (nonlinear) GMM parameter estimations over the various subsamples. Because a great number of asset pricing models will be tested, computational efficiency has strong appeal.

One may wonder by now why we focus exclusively on tests having a single break point as alternative. Surely, there are many other types of structural instabilities, such as cases with several breaks or with gradual movements in the δ_{ik} parameters. Constructing tests against all possible types of instabilities is simply impossible both statistically and practically. Fortunately, however, the situation is not hopeless because the single unknown break point statistics have power against a large class of parameter instability patterns far beyond what appears explicitly as an alternative in equation (22). Therefore, examining (only) single break point tests goes a long way toward our goal.

Before turning to the results we provide some details about the GMM estimation procedures common to all the models. First, the instruments are fairly standard: in addition to a constant we have (1) the one-month T-bill, (2) the dividend yield of the CRSP value-weighted NYSE stock index, (3) a detrended stock price level, (4) a measure of the slope of the term structure, (5) a quality-related yield spread in the corporate bond market, and (6) a January dummy. Details of these series appear in Ferson and Korajczyk (1995) and in Harvey (1991). The data are monthly and cover a sample from January 1927 until January 1988. The standard deviations and J-statistics in a GMM procedure critically depend on the covariance estimator. Our results are based on the estimator proposed by Andrews and Monahan (1992), a procedure that appears to have the best sampling properties among those currently available. Finally, regarding the Sup LM test we should note that one has to specify a sample range over which to compute the supremum. In all our computations we set $\Pi = [0.2T, 0.8T]$. The choice of 20 percent trimming is motivated by the length of the sample.

B. Stable Factors in the Conditional CAPM

In Panel A of Table I we report empirical results of the conditional CAPM described in equations (17) for the 12 industry-based portfolios. Panel B pertains to model (18), and we refer to it as explicit beta model because β_{it} is estimated as $Z_t\beta_c$. The model in Panel A is called the implicit beta model. The first row reports the J-statistic. The remaining seven rows in the table report the Sup LM test for parameter stability, each row representing an instrumental variable. Because two parameters are associated with each instrument in the implicit beta conditional CAPM, one appearing in the vector δ_M, the other in δ_i, we have a joint test for the two parameters associated with each instrument. For Panel B of Table I the statistics apply to three parameters, as an element of β_c is added with each instrument. Moreover, there are no J-statistics reported in Panel B as model (18) is just identified. Table II has exactly the same structure, except that results for size-sorted portfolios are considered.

Table I

Stable Time-Varying Beta Models with the Conditional CAPM: Industry-Based Classification

Panel A reports Implicit Beta models and Panel B reports Explicit Beta Models described in Section II. The set of instruments are listed in the left column of the table and described below. The data are the same as in Ferson and Korajczyk (1995) and cover monthly observations from January 1927 until January 1988. T-bill is the return on a one month T-bill, Div. yield is the dividend yield of the CRSP-weighted NYSE stocks, Mark. port. is the detrended stock price level, Maturity spr. is a measure of the slope of the term structure, Risk spr. is a quality-related yield spread in the corporate bond market, and January is a dummy variable set to one if the month is January. The implicit beta model is described by equation (17) in Section II, the explicit beta model by equation (18). The portfolios are NYSE value-weighted and industry-based. The entries to the tables are Sup LM tests for structural change. Their critical values appear in Andrews (1993, Table 1). Rejections at 10 percent appear in the table with *, at 5 percent with **, and at 1 percent with ***. The J-test is the overidentifying restrictions test, which is chi-squared with 7 degrees of freedom.

	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
Panel A: Implicit Beta Function Models												
J-test	8.4561	15.5556**	9.5685	9.9693	18.4212***	8.2977	5.1135	7.5999	13.7418*	8.7552	17.4164***	8.4974
Constant	5.6538	8.5861	12.0835**	19.0129***	5.6658	9.2023	13.0542**	13.7279**	11.0511*	8.7315	9.2351	10.1079*
T-bill	6.3883	9.6442	10.2414*	17.2727***	6.0636	11.5441*	10.6811*	20.2741***	13.8775**	10.0913*	8.5465	10.1274*
Div. yield	6.4314	8.9773	10.0793*	18.6618***	5.9701	9.8392	11.8432**	15.8659***	12.6606**	9.5095	9.3770	10.1865*
Mark. port.	5.6628	8.4872	11.1824*	18.2113***	4.3109	7.4730	10.5495*	12.0460**	10.1415*	6.9106	8.2083	8.8893
Maturity spr.	6.0368	5.1952	10.2652*	17.1359***	4.1179	8.6059	12.3346**	14.6369**	9.0326	7.7505	6.3391	9.1775
Risk spr.	5.3218	6.6729	7.5415	17.0632***	3.0120	5.7025	9.6299	10.3667*	13.2881**	5.3273	5.4790	7.4039
January	5.9531	9.1815	5.2715	7.2467	5.8469	7.9300	9.0587	6.2666	6.0237	5.2531	6.8119	3.5347
Panel B: Explicit Beta Function Models												
Constant	19.7776***	18.9413***	21.2024***	18.3620***	22.9432***	33.8775***	20.5167***	34.2726***	14.8246**	19.2092***	23.9346***	23.2181***
T-bill	18.7012***	16.9225**	24.2818***	16.2218**	23.5712***	36.5560***	19.7697***	32.6676***	12.7614*	22.0192***	27.0379***	26.5487***
Div. yield	18.6164***	19.4638***	21.2939***	15.9437**	23.0302***	34.1389***	15.7053**	33.8711***	13.1838*	19.9745***	22.0446***	22.0650***
Mark. port.	20.2345***	18.9488***	22.7727***	18.6820***	24.1310***	34.3247***	18.9904***	35.1880***	15.4509**	20.5434***	26.3213***	25.5308***
Maturity spr.	22.4618***	18.9465***	22.0660***	15.3660**	21.4682***	35.3595***	18.6272***	32.4882***	13.1189*	19.7394***	23.3330***	21.7168***
Risk spr.	20.6617***	14.3012**	21.6654***	18.4015***	19.5540***	32.2277***	22.0154***	29.5819***	14.1878**	19.5155***	20.9934***	22.2717***
January	17.5061**	9.3950	8.8273	10.0608	12.5905*	19.0430***	11.9949	22.4012***	10.9237	18.0111***	14.9813**	19.4349***

Table II

Stable Time-Varying Beta Models with the Conditional CAPM: Size-Sorted Portfolios

Panel A reports Implicit Beta models and Panel B reports Explicit Beta Models described in Section II. The set of instruments are listed in the left column of the table and described below. The data are the same as in Ferson and Korajczyk (1995) and cover monthly observations from January 1927 until January 1988. T-bill is the return on a one month T-bill, Div. yield is the dividend yield of the CRSP-weighted NYSE stocks, Mark. port. is the detrended stock price level, Maturity spr. is a measure of the slope of the term structure, Risk spr. is a quality-related yield spread in the corporate bond market, and January is a dummy variable set to one if the month is January. The implicit beta model is described by equation (17) in Section II, the explicit beta model by equation (18). The portfolios are NYSE value-weighted and industry-based. The entries to the tables are Sup LM tests for structural change. Their critical values appear in Andrews (1993, Table 1). Rejections at 10 percent appear in the table with *, at 5 percent with **, and at 1 percent with ***. The *J*-test is the overidentifying restrictions test, which is chi-squared with 7 degrees of freedom.

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
Panel A: Implicit Beta Function Models										
J-test	8.5568	11.3002	12.1601*	11.0763	11.5052	12.0629*	12.8190*	10.8563	12.5419*	5.4799
Constant	10.7347*	11.3862*	8.6776	8.2499	9.5281	12.4411**	8.9419	11.6351*	8.6630	16.8273***
T-bill	10.1020*	10.8108*	13.4065**	11.3458*	10.9153*	12.8170**	11.3349*	14.9146**	10.5843*	13.8874**
Div. yield	11.3238*	11.7282*	9.9046	8.8673	9.7129	12.5172**	9.4798	12.5863**	9.0066	17.1910***
Mark. port.	9.9315	10.4829*	8.3360	7.7991	8.3102	10.6512*	7.8064	10.0998*	6.7923	16.0009***
Maturity spr.	8.7033	9.5010	8.3399	7.9619	7.7540	12.1721**	8.4567	9.9241	10.0226*	15.6434***
Risk spr.	9.4851	9.6367	7.2096	7.1454	6.4412	10.7859*	6.0006	6.2108	6.1244	13.9341**
January	5.5833	5.4106	10.6941*	11.4727*	6.5779	9.6168	11.3160*	10.0612*	5.3646	8.2822
Panel B: Explicit Beta Function Models										
Constant	8.5568	11.3002	12.1601*	11.0763	11.5052	12.0629*	12.8190*	10.8563	12.5419*	5.4799
T-bill	10.7347*	11.3862*	8.6776	8.2499	9.5281	12.4411**	8.9419	11.6351*	8.6630	16.8273***
Div. yield	10.1020*	10.8108*	13.4065**	11.3458*	10.9153*	12.8170**	11.3349*	14.9146**	10.5843*	13.8874**
Mark. port.	11.3238*	11.7282*	9.9046	8.8673	9.7129	12.5172**	9.4798	12.5863**	9.0066	17.1910***
Maturity spr.	9.9315	10.4829*	8.3360	7.7991	8.3102	10.6512*	7.8064	10.0998*	6.7923	16.0009***
Risk spr.	8.7033	9.5010	8.3399	7.9619	7.7540	12.1721**	8.4567	9.9241	10.0226*	15.6434***
January	9.4851	9.6367	7.2096	7.1454	6.4412	10.7859*	6.0006	6.2108	6.1244	13.9341**

Let us focus first on the implicit beta conditional CAPM appearing in Panels A of Tables I and II. It was noted that the *J*-statistic is a diagnostic test ill-equipped to scrutinize a model in terms of its structural invariance and by the same token the ability of a model to predict the market price for risk. Although the results in Tables I and II are not as striking as those that will be reported below, they do reveal some quite extreme cases. We start with two such cases: Industry 8 (Transportation) in Table I and the portfolio of returns on the largest firms (Size 10) in Table II. In both cases, according to the overidentifying restrictions tests, the conditional CAPM is not rejected. Yet, for all instruments except the January dummy there is strong evidence of parameter instability. Hence, despite the favorable evidence according to the usual *J*-statistic, it is clear that the return on both portfolios cannot be satisfactorily priced with the conditional CAPM. Although both cases together with Industry 4 (Basic Industries) are extreme, it is clear that Tables I and II contain many other examples. In fact, in only one case (Industry 1, Petroleum) of the 22 asset return series is there no rejection of the conditional CAPM with the *J*-statistic nor with the Sup LM tests. If we were to rely only on the *J*-statistic we would not reject the conditional CAPM for 6 of the 12 industries and for 2 of the 10 size-based portfolios.

The first of three empirical examples underscores several important points that motivate our study. We report a set of models that would be found empirically acceptable, according to their overidentifying restrictions, for explaining the returns on selected portfolios with a pricing formula based on a set of common instruments. After eliminating all the cases where structural breaks are found we are left with only one case. In practical terms this means these instruments do not yield a satisfactory dynamic conditional asset pricing model. In such circumstances either the model needs to be modified or we need to search for a stable risk factor alternative. Before making any conclusions on this we turn to the other three models discussed in Section II to appraise their performance.

The first is the conditional CAPM with the explicit beta model $\beta_{it} = Z_t \beta_{ic}$ that appears in equation (18). The results are reported in Panels B of Tables I and II. We find resounding rejections of the stability hypothesis. One may wonder what happened while moving from equation (17) to equation (18). Note that the Sup LM tests now apply to three parameters instead of just the two associated with each instrument. The added parameter belongs to β_{ic}. The overwhelming rejections reveal that the linear representation $Z_t \beta_{ic}$ is totally inadequate. Consequently the simplification, which avoids the wieldy specification of the covariance/variance ratio, is simply not acceptable. Comparison of Panels A and B of both tables underlines again the scope of testing for structural invariance of parameters to uncover misspecification.

Because we report evidence for structural breaks it is worth mentioning that the estimated break points, that is, the sample points where the supremum of the LM statistics is attained, do *not* occur at the same moment; they are in fact quite dispersed. It should first be noted that it is not clear that the supremum is attained at the true break point (formal proofs for the general case of nonlinear dynamic system of equations are to our knowledge nonexistent). In the event they yield an unbiased estimation of break points we must conclude that no specific event caused the rejection of parameter stability.

C. Stable Factors in the Conditional Multifactor APT

To describe the empirical results let us return to equation (19) and recall the interpretation of each of the parameters. There are essentially three sets of moment conditions, the first two define conditional expectations (linear projections) of asset returns and the third relates to the multifactor beta model. Hence, breaks in the parameters δ_i and γ_i reflect a misspecification of the statistical models for the predictable dynamics in returns or factor mimicking portfolios. In contrast, the hypothesis of fixed conditional betas is more a fundamental and crucial assumption from an asset pricing perspective (see, e.g., Ferson (1990, Table VIII) on this issue). We will first discuss the two sets of projection equations and then turn to the risk-pricing equation. We begin with the empirical results regarding the stability of the coefficients δ_i in equation (19) obtained from projecting the six instruments plus constant on size-sorted and industry-based portfolio returns.

Before discussing the Sup LM test results we need to be more specific about the specification of the factors in the conditional APT model. Two alternative sets of risk factors are examined. The first set consists of economic variables similar to those of Chen, Roll, and Ross (1986) and Ferson and Harvey (1991). Among those factors is the market return measured by the S&P 500. The latter is also the market portfolio for the conditional CAPM. Mimicking portfolios are constructed using individual common stocks for the five factors. The second approach is motivated by many previous studies of the APT and uses the asymptotic principal components method of Connor and Korajczyk (1989) to estimate the common factors. We compute the results for both factor configurations but report only the economic factors. We do not report explicitly the results with the principal component factors because they are quite similar except that one typically finds even more evidence for structural breaks.

Tables III and IV cover the empirical results for the conditional APT with a combination of industry-based and size-sorted portfolios. The J-statistics appear at the top of each table and the Sup LM statistics are listed in the rows labeled δ_{all} and $\delta_j, j = 1,\ldots,7$.[9] The tests corresponding to δ_{all} are joint tests for all seven instruments (the first being a constant); the others measure each instrument individually. Let us focus on the results in Table III, which cover the 12 industries selected by Ferson and Korajczyk. According to the J-statistic we would almost never reject the model.

[9] To simplify the notation in Tables III and IV we should note that the index j to δ_j is not to be confounded with index δ_i in (19). The latter refers to asset i and represents the entire vector (δ_{all} in the tables), but δ_j is an element of δ_{all}.

Table III

Stable Factors Structures in the Conditional APT: Industry Classification with Economic Variables Factors

The J-test is chi-squared with 30 degrees of freedom. The entries to the table are Sup LM tests. Their critical values appear in Andrews (1993, Table 1). δ_{all} tests for all coefficients of δ together (7 parameters), δ_i ($i = 1,2,\ldots,7$) tests for all coefficients of δ one by one (1 parameter each). γ_{ic} ($i = 1,2,\ldots,5$) tests for all coefficients of γ column by column (7 parameters by column). β_{all} tests for all coefficients of β together (5 parameters), β_i ($i = 1,2,\ldots,5$) tests for all coefficients of β one by one (1 parameter each). T-bill is the return on a one month T-bill, Div. yield is the dividend yield of the CRSP-weighted NYSE stocks, Mark. port. is the detrended stock price level, Maturity spr. is a measure of the slope of the term structure, Risk spr. is a quality-related yield spread in the corporate bond market, and January is a dummy variable set to one if the month is January. The Conditional APT model is described by equation (19) in Section II. Rejections at 10 percent appear in the table with *, at 5 percent with **, and at 1 percent with ***.

	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
J-test	36.7	35.2	27.8	36.5	31.4	37.7	27.5	48.8**	34.6	26.2	49.8**	33.5
						Sup LM						
δ_{all}	16.3	15.0	9.0	17.9	19.6	10.3	12.6	14.2	16.6	13.8	14.2	13.5
δ_1	1.7	0.6	5.1	1.5	10.0**	2.3	1.9	1.8	4.3	7.7*	2.1	1.8
δ_2	3.2	1.5	2.6	1.3	3.9	1.0	2.1	1.8	5.5	6.6	3.6	2.5
δ_3	3.9	1.2	4.9	1.5	12.0**	1.4	1.7	1.3	3.7	5.7	2.6	2.6
δ_4	3.4	0.5	4.9	1.4	9.9**	1.6	1.7	1.6	4.2	7.0	2.0	2.0
δ_5	5.8	2.5	3.2	2.9	7.3*	2.8	2.1	3.6	4.3	5.5	1.5	3.4
δ_6	8.3*	1.8	3.2	5.7	13.0***	3.0	3.3	1.4	1.8	7.7*	2.7	3.2
δ_7	3.5	1.2	1.8	2.1	5.6	3.9	6.5	1.9	2.4	4.3	6.7	3.0
γ_{1c}	77.9***	76.4***	66.9***	62.6***	99.2***	69.5***	62.7***	62.5***	61.4***	81.6***	64.8***	71.7***
γ_{2c}	170.6***	112.0***	108.4***	95.7***	142.0***	95.8***	91.9***	92.5***	90.4***	131.3***	105.2***	97.9***
γ_{3c}	32.2***	27.8***	15.4	23.5**	22.9**	21.0*	12.7	33.7***	22.7**	17.6	25.4**	16.2
γ_{4c}	31.0***	23.0**	51.2***	19.3	17.9	20.0*	15.7	37.3***	23.2**	14.5	18.4	19.1
γ_{5c}	43.1***	16.1	9.0	16.0	20.5*	12.6	9.8	27.0***	9.7	20.6*	14.9	17.9
β_{all}	36.8***	17.6*	7.1	8.7	39.4***	22.2**	8.4	17.0*	17.8*	9.0	28.1***	19.8**
β_1	10.3**	7.8*	2.1	5.5	1.0	2.9	3.1	3.9	4.1	2.4	1.6	9.4
β_2	10.7**	9.1**	1.0	2.5	5.5	3.6	3.0	5.0	2.0	4.8	3.1	16.2***
β_3	1.2	3.6	0.8	2.8	24.9***	3.9	3.3	2.0	5.8	2.5	19.9***	2.2
β_4	4.2	14.5***	1.1	4.2	4.2	0.8	0.8	2.8	8.5*	1.7	5.8	0.7
β_5	18.2***	3.4	1.7	1.9	25.5***	11.0**	5.0	14.3***	10.7**	3.9	18.3***	4.4

Table IV

Stable Factors Structures in the Conditional APT: Size Classification with Economic Variables Factors

The J-test is chi-squared with 30 degrees of freedom. The entries to the table are Sup LM tests. Their critical values appear in Andrews (1993, Table 1). δ_{all} tests for all coefficients of δ together (7 parameters), δ_i ($i = 1,2,\ldots,7$) tests for all coefficients of δ one by one (1 parameter each). γ_{ic} ($i = 1,2,\ldots,5$) tests for all coefficients of γ column by column (7 parameters by column). β_{all} tests for all coefficients of β together (5 parameters), β_i ($i = 1,2,\ldots,5$) tests for all coefficients of β one by one (1 parameter each). T-bill is the return on a one month T-bill, Div. yield is the dividend yield of the CRSP-weighted NYSE stocks, Mark. port. is the detrended stock price level, Maturity spr. is a measure of the slope of the term structure, Risk spr. is a quality-related yield spread in the corporate bond market, and January is a dummy variable set to one if the month is January. The Conditional APT model is described by equation (19) in Section II. Rejections at 10 percent appear in the table with *, at 5 percent with **, and at 1 percent with ***.

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
J-test	29.3	32.3	46.9**	53.5***	50.7**	46.5**	39.5	37.4	30.1	54.9***
					Sup LM					
δ_{all}	13.2	11.1	14.0	10.9	16.4	19.1	31.7***	23.2**	14.3	40.7***
δ_1	3.2	8.2*	5.5	8.4*	10.5**	4.1	4.8	5.8	3.9	2.4
δ_2	1.7	2.4	6.5	4.3	9.6**	1.7	10.1**	7.9*	11.2**	3.8
δ_3	3.4	10.0**	5.5	9.5**	11.7**	5.1	4.9	4.0	2.6	3.0
δ_4	2.5	8.4*	4.8	8.2*	10.2**	4.0	3.2	4.5	4.0	3.0
δ_5	1.0	3.9	5.8	5.4	8.9**	3.4	10.0**	4.0	10.5**	10.5**
δ_6	3.8	7.7*	2.9	7.4*	6.6	8.9*	5.8	2.0	2.6	2.1
δ_7	5.0	3.0	3.0	2.6	4.9	1.9	1.3	4.9	3.4	3.1
γ_{1c}	64.3***	59.7***	65.1***	61.7***	64.4***	75.4***	66.5***	57.2***	67.5***	75.2***
γ_{2c}	103.0***	105.0***	109.7***	115.0***	141.6***	126.4***	110.4***	98.8***	108.1***	108.9***
γ_{3c}	31.3***	26.0**	13.9	19.9*	30.0***	24.0**	19.7*	17.0	23.5**	26.2**
γ_{4c}	33.3***	28.8***	14.6	23.4**	25.5**	27.5***	24.1**	18.8	24.4**	22.0**
γ_{5c}	12.2	8.2	9.7	10.8	13.8	17.2	29.1***	22.8**	12.5	43.2***
β_{all}	8.6	9.5	7.3	16.2	13.1	18.5**	18.6**	17.5*	18.1*	39.2***
β_1	2.8	2.2	5.2	3.7	2.2	6.1	8.6*	3.2	2.3	2.7
β_2	4.6	4.3	5.3	3.3	2.4	7.5*	6.6	3.2	2.7	6.2
β_3	1.0	4.7	4.6	2.3	5.9	4.9	10.9**	6.0	4.7	25.4***
β_4	2.5	1.7	1.3	6.3	4.7	8.5*	3.7	10.5**	12.9***	7.1
β_5	4.9	2.4	1.6	3.1	3.4	4.6	11.8**	4.6	4.9	38.0***

Based on the Sup LM tests for δ_{all} and the individual δ_i we find very little evidence for breaks among the industry-based portfolios (Table III) except for Industry 5 (Food/Tobacco) but much more evidence for breaks in the size-based classification (Table IV). The situation is quite different with the parameters in the second block of moment conditions, where there is very strong evidence against the null hypothesis.

The second set of moment conditions in equation (19), like the first, involves projections on the set of instruments described before, to extract the predictable part of the $K \times 1$ vector F_t. Because this is a multivariate process prediction with $K = 5$ we focus on tests for each column which project the entire set of instruments on each of the five factor-mimicking portfolios. Hence, we use the notation γ_{ic}, $i = 1,\ldots,5$, to denote the tests associated with each of the column vectors.[10] The overwhelming evidence of breaks in γ_{ic} means it is very difficult to predict the returns on the factor-mimicking portfolios.

So far we have discussed only those parameters related to the two blocks of projection equations for the portfolio return and the factors. The remaining coefficients pertaining to the conditional betas are more important from an asset pricing point of view. We find in many cases strong evidence of instability, notably in Industries 1 (Petroleum), 2 (Finance/Real Estate), 5 (Food/Tobacco), 6 (Construction), 8 (Transportation), 11 (Services), 12 (Leisure), and to a lesser extent Industry 9 (Utilities). For the size-sorted portfolios in Table IV we seem to reject constant beta entries particularly for the large sizes of NYSE stocks.

D. Stable Factors in the Nonlinear APT

Last we examine the nonlinear APT proposed by Bansal et al. (1993). As in Section III.B with Harvey's model, we do not attempt to exactly replicate their data and estimates. Instead, for comparison, we use the Ferson and Korajczyk (1995) data set of sized-sorted and industry-based portfolios to estimate the nonlinear APT specified in equations (14) and (16) using the same set of instruments. This means we have seven instruments, including a constant, to specify the moment conditions in equation (16). Because there are five parameters in equation (16) we have two overidentifying restrictions. The results are reported in two tables, one covering the asset returns for each of the 10 size-sorted portfolios and the other containing the industry-based portfolios.

The following tests appear in Tables V and VI: (1) tests for the stability of each of the five parameters in the nonlinear APT separately, (2) two joint tests, one involving the parameters of the "nonlinear part," namely β_{2M} and β_{5M}, and one involving the joint set of five beta parameters. The results in Table V show that there are clearly problems with the small size categories. All other size categories appear to be well fitted by a stable nonlinear APT

[10] We could not perform an overall test for the entire matrix γ involving 35 coefficients as no critical values are available for that many coefficients. For reason of space we do not report individual tests nor tests associated with a particular instrument in this case.

Table V

Stable Factors Structures in Nonlinear APT: Size Classification

The Nonlinear APT is described by equations (14) and (16) in Section II. The instruments are the same as for the Conditional APT, namely (1) the return on a one month T-bill, (2) the dividend yield of the CRSP-weighted NYSE stocks, (3) the detrended stock price level, (4) a measure of the slope of the term structure, (5) a quality-related yield spread in the corporate bond market, (6) a dummy variable set to one if the month is January, and (7) a constant. The entries to the table are Sup LM tests where β_i represents tests for each parameter separately. β_{2M} & β_{5M} tests these two parameters jointly. β_{all} tests all parameters together. Rejections at 10 percent appear in the table with *, at 5 percent with **, and at 1 percent with ***.

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
β_0	9.3111**	7.3400*	3.2233	3.0829	2.5073	1.5516	1.5604	1.1807	2.8372	6.1427
β_{1f}	8.7747*	6.5922	2.5417	2.2985	1.9833	1.7167	2.0838	1.6714	2.2071	1.9955
β_{1M}	3.5560	5.6185	2.3653	1.5622	1.7633	1.5459	1.6419	2.5816	1.2021	1.0837
β_{2M}	10.7844**	3.5330	2.0178	1.3234	1.1955	0.6883	0.5087	0.2833	0.4037	0.2519
β_{5M}	10.4911**	3.2251	1.3729	0.9411	1.1827	0.8951	0.9466	0.6221	1.2187	1.0033
β_{2M} & β_{5M}	12.2474**	7.1412	5.6991	2.0473	2.8680	2.7284	4.4392	3.6401	3.6456	3.4788
β_{all}	31.3382***	27.0550***	7.4364	5.5230	4.4484	5.0572	6.2892	9.0184	7.8746	14.5524

Table VI

Stable Factors Structures in Nonlinear APT: Industry Classification

The Nonlinear APT is described by equations (14) and (16) in Section II. The instruments are the same as for the Conditional APT, namely (1) the return on a one month T-bill, (2) the dividend yield of the CRSP-weighted NYSE stocks, (3) the detrended stock price level, (4) a measure of the slope of the term structure, (5) a quality-related yield spread in the corporate bond market, (6) a dummy variable set to one if the month is January, and (7) a constant. The entries to the table are Sup LM tests where β_i represents tests for each parameter separately. β_{2M} & β_{5M} tests these two parameters jointly. β_{all} tests all parameters together. Rejections at 10 percent appear in the table with *, at 5 percent with **, and at 1 percent with ***.

	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
β_0	8.6411*	4.0442	3.4443	10.5322**	18.2567***	4.4049	1.1633	3.1189	8.4750*	2.1844	5.6066	2.4060
β_{1f}	3.1632	2.0110	1.7122	4.3337	24.6479***	11.3713**	1.2177	1.1837	5.2264	7.2989*	1.3148	1.7611
β_{1M}	5.8645	2.8783	1.2302	1.4042	15.0796***	5.0320	1.3047	2.6226	4.7729	2.5026	2.6740	3.4647
β_{2M}	19.5888***	1.8722	0.5913	2.6989	7.7502*	7.9964*	0.7180	4.6802	4.2980	5.0687	5.9712	5.7579
β_{5M}	18.1689***	1.9036	0.9277	3.4786	6.5603	6.9189	0.9197	4.9553	3.3244	4.6954	5.2320	5.5474
β_{2M} & β_{5M}	19.6123***	2.1695	2.0559	4.1190	7.8974	8.4659	2.6530	5.0061	5.0703	5.5203	6.1968	5.7582
β_{all}	26.1288***	9.6461	11.0329	18.6607**	37.3194***	16.8289*	9.7900	13.7336	12.2965	10.3140	13.6920	9.2915

model. This is far better than the conditional APT of the previous section, because for almost all portfolios the model seems acceptable. It only fails to explain returns on very small firms which probably are more affected by informed trading and idiosyncratic events. The nonlinear APT also appears quite successful if one looks at industry-based portfolios. In Table VI we can see that for at least half of the 12 industries there is no instability according to the Sup LM tests. The industries where the model fails are: Industry 1 (Petroleum), 4 (Basic Industries), 5 (Food and Tobacco), 6 (Construction), and to some extent perhaps 9 (Utilities) and 10 (Services).

IV. Do Time-Varying Betas Help or Hurt?

The empirical results in the previous section revealed that the time-varying beta conditional CAPM and APT models do not seem to capture very well the temporal dynamics of betas. Consequently, they misprice risk. Is this mispricing serious, so serious that we are bound to make larger errors in comparison to fixed beta models? This is a question of relative mispricing of one (misspecified) model against another one. To address this we will compute the in-sample root mean squared error (RMSE) of the conditional CAPM and APT models appearing in equations (17) and (19), respectively, and compare them with the RMSE of the fixed beta model, specified through the following moment conditions[11]:

$$E\begin{pmatrix} r_{Mt+1} - Z_t\delta_M \\ r_{t+1} - \beta Z_t\delta_M \end{pmatrix} \otimes Z_t = 0. \tag{23}$$

Note that the model in equation (23) uses the expected return $E(r_{Mt+1}|Z_t)$ instead of the actual return as a factor so that it involves the same conditioning information set Z_t as the conditional CAPM and APT and has no informational advantage that would make comparisons of forecasts difficult.[12]

[11] The RMSE calculations for the conditional CAPM and APT are based on the pricing error as defined in Ferson and Harvey (1993), equation (8). Note that we report results for the explicit beta function conditional CAPM. The results for the model in equation (17) are similar to those obtained with equation (18) and are therefore omitted.

[12] The model in equation (23) corresponds to equation (7) in Harvey (1991). It should be noted that the comparison of root mean squared errors does *not* involve the nonlinear APT. Indeed, although the results in the previous section are favorable with regard to the stability of the pricing kernel, they do not yield straightforwardly a prediction model. This problem arises in any approach that directly tries to specify a pricing kernel such as consumption-based asset pricing methods (e.g., see Hansen and Singleton (1982) or Epstein and Zin (1991)). The main reason we do not engage in a comparison involving the nonlinear APT is that the model needs to be *augmented* in nontrivial ways with prediction formulas to generate predictions of returns. Such augmentation would involve quite a number of auxiliary assumptions and novelties that would take us far beyond the scope of the present paper. We are grateful to S. Viswanathan for having given thought to several of our queries regarding prediction formula for nonlinear APT models.

We consider the RMSE of pricing errors computed from three models: (1) the unconditional CAPM, (2) the conditional CAPM, and (3) the conditional APT using economic factors. These comparisons are performed for the 10 size-based portfolios and the 12 industry-based returns.

Table VII summarizes the empirical results. For industry-based portfolios, the data show that the unconditional CAPM dominates all other specifications quite often. For Industries 1, 5, 6, 10, 11, and 12, or 6 of 12 industries, it has the smallest RMSE. Moreover, there is often only a marginal difference for industries where the unconditional CAPM does not outperform the conditional models.

Among the conditional models we observe that the conditional CAPM is often the best among all the conditional models when the Morgan Stanley Capital International (MSCI) index is taken as the market portfolio instead of the S&P 500. This model is included in the comparison to point us to a clearer interpretation of our results. Indeed, a plausible explanation for the results in Table VI is that betas change through time very slowly. The conditional APT and CAPM models may have a tendency to overstate the time variation and as a result produce beta risk that is too volatile and changing too rapidly. The evidence reported in Braun, Nelson, and Sunier (1995) confirms this interpretation of our results. Indeed, they report very weak time variation in direct estimates of conditional betas obtained from ARCH-type specifications. Hence, they find that although equity market returns are volatile and conditionally heteroskedastic, the betas are only modestly time varying and hence not very volatile. The conditional CAPM and APT models discussed in this paper use volatile market returns and other instruments to model the slowly changing beta risk. This also explains why replacing the S&P 500 by the MSCI index in the conditional CAPM sometimes improves the model dramatically. Indeed, turning our attention to Table VIII where the standard errors of the sample beta risk obtained from the two conditional CAPM specifications are compared, we observe that beta risk as a function of the S&P 500 is much more volatile than when the MSCI is used. A major reason (although not the only one) is that the MSCI index is less volatile. The bottom panels of Tables VII and VIII show that the size results are similar to the industry results given in the top panels.

V. Conclusion

There is a general consensus that the static CAPM is unable to explain the expected returns on stocks. The empirical study of Fama and French (1992) has indeed fueled a lively debate and prompted the pursuit of some form of conditional models. We take several APT-type models of recent vintage which have as their key ingredient a time-varying structure in factor-return trade-offs. Those models are at the same time sophisticated and fragile. They are sophisticated because they exploit dynamics in predictability and/or nonlinearities. But they are also fragile because they must deal with time-varying betas and are therefore more prone to sources

Table VII

Root Mean Squared Errors of Pricing Errors for CAPM, Conditional CAPM, and APT Models

The Unconditional CAPM is described by equation (23) in Section IV, the Conditional CAPM by equation (17) in Section II, and the Conditional APT by equation (19) also in Section II. The instruments are (1) the return on a one month T-bill, (2) the dividend yield of the CRSP-weighted NYSE stocks, (3) the detrended stock price level, (4) a measure of the slope of the term structure, (5) a quality-related yield spread in the corporate bond market, (6) a dummy variable set to one if the month is January, and (7) a constant. The entries to the table are the Root Mean Squared Errors for the unconditional CAPM, the conditional CAPM and the conditional APT. The calculations are in-sample covering January 1927 to January 1988 when the S&P 500 index is used and February 1970 to January 1988 for the Morgan Stanley Capital International (MSCI).

	Industry-Based Portfolio Classifications											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Unconditional CAPM (SP500)	0.0450	0.0427	0.0510	0.0448	0.0343	0.0492	0.0470	0.0530	0.0332	0.0439	0.0536	0.0531
Conditional CAPM - Explicit Beta Function (SP500)	0.0814	0.0965	0.0754	0.0955	0.1385	0.0662	0.1011	0.0656	0.1385	0.1014	0.1351	0.1051
Conditional CAPM - Explicit Beta Function (MSCI)	0.0485	0.0405	0.0466	0.0376	0.0354	0.0511	0.0456	0.0500	0.0306	0.0535	0.0673	0.0578
Conditional APT - Economic Factors	0.0458	0.0425	0.0514	0.0444	0.0340	0.0493	0.0469	0.0531	0.0329	0.0440	0.0540	0.0535

	Size-Based Portfolio Classifications									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Unconditional CAPM (SP500)	0.0723	0.0628	0.0573	0.0550	0.0523	0.0515	0.0479	0.0455	0.0439	0.0380
Conditional CAPM -Explicit Beta Function (SP500)	0.0964	0.0722	0.0602	0.0632	0.0537	0.0566	0.0552	0.0455	0.0516	0.0384
Conditional CAPM - Explicit Beta Function (MSCI)	0.0677	0.0647	0.0589	0.0567	0.0557	0.0503	0.0457	0.0431	0.0406	0.0343
Conditional APT - Economic Factors	0.0723	0.0629	0.0574	0.0550	0.0527	0.0515	0.0480	0.0454	0.0438	0.0383

Table VIII

Sample Standard Errors of Betas in Conditional CAPM

The Conditional CAPM is described by equation (17) in Section II. The instruments are (1) the return on a one month T-bill, (2) the dividend yield of the CRSP-weighted NYSE stocks, (3) the detrended stock price level, (4) a measure of the slope of the term structure, (5) a quality-related yield spread in the corporate bond market, (6) a dummy variable set to one if the month is January, and (7) a constant. The entries to the table are the standard errors of the sample beta risk obtained from the Conditional CAPM. The calculations cover a sample from January 1927 to January 1988 when the S&P 500 index is used and February 1970 to January 1988 for the Morgan Stanley Capital International (MSCI).

	Industry-Based Portfolio Classifications											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
SP500	8.8173	10.6645	6.3569	10.3632	16.5749	5.1314	10.9083	4.4994	16.5831	10.8733	14.0721	10.9478
MSCI	2.1085	1.2341	1.6389	0.7398	1.1356	1.6023	1.7213	1.3275	0.9143	2.4924	4.0254	2.6685

	Size-Based Portfolio Classifications									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
SP500	6.8063	3.2454	1.5740	3.0291	0.9446	2.3254	2.8158	0.6537	2.9634	1.0138
MSCI	2.9091	3.3814	3.0575	2.8819	2.7384	2.2673	1.8547	1.5387	1.4238	0.6057

of misspecification and hence mispricing of assets. The role of this paper is to show (1) how serious the problem of parameter instability is and, most importantly, (2) that despite all the efforts to model time-varying beta risk we find that constant beta models in many cases still yield on average better predictions. Many have argued convincingly that the static CAPM should be replaced by some form of conditional CAPM. Our paper shows that the search for a satisfactory specification is still far from accomplished. It suggests that more attention should be given to the consequences of using the "wrong" CAPM (see for instance Ferson and Locke (forthcoming)). Our findings also suggest, however, that the recently proposed class of nonlinear APT models appear surprisingly stable in comparison with the conditional CAPM and APT models and therefore show the greatest promise for further development.

REFERENCES

Andrews, Donald W. K., 1993, Tests for parameter instability and structural change with unknown change point, *Econometrica* 61, 821–856.

Andrews, Donald W. K., and John C. Monahan, 1992, An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator, *Econometrica* 60, 953–966.

Andrews, Donald W. K., and Werner Ploberger, 1994, Optimal tests when a nuisance parameter is present only under the alternative, *Econometrica* 62, 1383–1414.

Bansal, Ravi, David Hsieh, and S. Viswanathan, 1993, A new approach to international arbitrage pricing theory, *Journal of Finance* 48, 1719–1747.

Bansal, Ravi, and S. Viswanathan, 1993, No arbitrage and arbitrage pricing: A new approach, *Journal of Finance* 48, 1231–1262.

Braun, Philip A., David B. Nelson, and Alain M. Sunier, 1995, Good news, bad news, volatility and betas, *Journal of Finance* 50, 1575–1603.

Chamberlain, Garry, and Michael Rothschild, 1983, Arbitrage and mean variance analysis on large asset markets, *Econometrica* 51, 1281–1304.

Chen, Nai-fu, Richard Roll, and Stephen A. Ross, 1986, Economic forces and the stock market, *Journal of Business* 59, 383–403.

Connor, Gregory, 1984, A unified beta pricing theory, *Journal of Economic Theory* 34, 13–31.

Connor, Gregory, and Robert A. Korajczyk, 1989, An intertemporal beta pricing model, *Review of Financial Studies* 2, 373–392.

Dumas, Bernard, and Bruno Solnik, 1995, The world price of exchange rate risk, *Journal of Finance* 50, 445–480.

Epstein, Larry, and Stanley E. Zin, 1991, Substitution, risk aversion and the temporal behavior of consumption and asset returns II: An empirical analysis, *Journal of Political Economy* 99, 263–286.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Ferson, Wayne E., 1985, Changes in expected risk premiums and security risk measures, *Proceedings of the European Finance Association*.

Ferson, Wayne E., 1990, Are the latent variables in time-varying expected returns compensation for consumption risk?, *Journal of Finance* 45, 397–430.

Ferson, Wayne E., and Campbell R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.

Ferson, Wayne E., and Campbell R. Harvey, 1993, The risk and predictability of international equity returns, *Review of Financial Studies* 6, 527–566.

Ferson, Wayne E., and Robert A. Korajczyk, 1995, Do arbitrage pricing models explain the predictability of stock returns?, *Journal of Business* 68, 309–349.

Ferson, Wayne E., and Dennis H. Locke, Estimating the cost of capital thorugh time: An analysis of the sources of error, *Management Science*, forthcoming.

Ghysels, Eric, Alain Guay, and Alastair Hall, 1998, Predictive tests for structural change with unknown breakpoint, *Journal of Econometrics* 82, 209–234.

Ghysels, Eric, and Alastair Hall, 1990a, A test for structural stability of Euler conditions parameters estimated via the generalized method of moments estimator, *International Economic Review* 31, 355–364.

Ghysels, Eric, and Alastair Hall, 1990b, Are consumption-based intertemporal capital asset pricing models structural?, *Journal of Econometrics* 45, 121–139.

Hansen, Lars P., 1982, Large sample properties of generalized method of moments estimators, *Econometrica* 50, 1029–1054.

Hansen, Lars P., and Ravi Jaganathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225–262.

Hansen, Lars P., and Kenneth J. Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica* 50, 1269–1286.

Harvey, Campbell R., 1991, The world price of covariance risk, *Journal of Finance* 46, 111–157.

Harvey, Campbell R., 1995, Predictable risk and returns in emerging markets, *Review of Financial Studies* 8, 773–816.

Huang, Chi-fu, 1987, An intertemporal general equilibrium asset pricing model: The case of diffusion information, *Econometrica* 55, 117–142.

Huberman, Gur, 1982, A simple approach to arbitrage pricing theory, *Journal of Economic Theory* 28, 182–191.

Ingersoll, Jonathan, 1984, Some results in the theory of arbitrage pricing, *Journal of Finance* 39, 1021–1039.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The conditional CAPM and the cross-section of expected returns, *Journal of Finance* 51, 3–54.

Lehman, Bruce N., and David Modest, 1988, The empirical foundations of the arbitrage pricing theory, *Journal of Financial Economics* 21, 213–254.

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolio and capital budgets, *Review of Economics and Statistics* 47, 13–37.

Lucas, Robert E. Jr., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1426–1446.

Merton, Robert, 1973, An intertemporal asset pricing model, *Econometrica* 41, 867–880.

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.

Sharpe, William F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Stulz, René, 1981, A model of international asset pricing, *Journal of Financial Economics* 9, 383–406.