Is sector-neutrality in factor investing a mistake?*

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Abstract
Stock characteristics have two sources of predictive power. First, a characteristic might be valuable in identifying high or low expected returns across industries. Second, a characteristic might be useful in identifying individual stock expected returns within an industry. Past studies generally find that the firm-specific component is the strongest predictor, leading many to sector neutralize their factor exposures. We show that this problem is equivalent to the classic two risky-asset problem and derive the condition that decides when the sector component of a characteristic should be omitted. We show both analytically and empirically that the long–short investor is more likely to benefit from hedging out sector bets, whereas the long-only investor is more likely to benefit from investing in the factor as it stands.

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1 Introduction

Firm characteristics such as size, book-to-market ratio, and momentum are often correlated with expected returns (Banz (1981); Rosenberg, Reid, and Lanstein (1985); Fama and French (1993); Jegadeesh and Titman (1993)). The market-wide predictive power of these characteristics may stem from their industry component, their firm-specific component, or both. Moskowitz and Grinblatt (1999), for example, argue that momentum in stocks stems from the industry component. Other studies such as Asness, Porter, and Stevens (2000) and Novy-Marx (2013) find that the firm-specific component of characteristics contains most of the information, suggesting that an investor benefits from forming portfolios that neutralize sector exposures.

In this paper, we first confirm that the *within* (firm-specific) component of stock characteristics contains more information about the cross-section of expected returns than the *across* (sector) component. We then derive a condition that determines when the weaker component of a predictor should be omitted. Using aggregate values for Sharpe ratios and correlation coefficients we predict that this condition—which identifies whether the across-sector component is redundant—will be met frequently in long–short portfolios; therefore, the long–short investor likely gains from sector neutralizing. In contrast, the same condition is unlikely to hold in long-only portfolios. Therefore, long-only factor performance is more likely to degrade from sector neutralizing. Empirical bootstraps of historical data of factors, constructed using various portfolio construction techniques, show that our analytical results are accurately reflected in the actual data.

To illustrate, assume that signal Z consists of two parts, $Z = X + Y$. If Z is the book-to-market ratio, then X would be its within-sector component and Y the sector component. The investor can use the signal as it stands and earn a return of $Z \times r = (X + Y) \times r$, or she can use

X or Y independently to earn $X \times r$ or $Y \times r$. If X predicts returns more accurately than Y, then the risk-return profile of $X \times r$ dominates that of $Y \times r$. Should the mean-variance investor form portfolios based on X or $X + Y$? This problem, the redundancy of predictor Y in the presence of X, is equivalent to a static two risky-asset problem with the solution,

$$\frac{SR_Y}{SR_X} \leq \rho, \tag{1}$$

where S_Y and S_X denote the Sharpe ratios of the resulting portfolios, $Y \times r$ and $X \times r$, respectively, and ρ is the correlation coefficient between the two portfolios. The inequality in (1) states that the trade-off between diversification benefits and mean-variance efficiency commands the relative value of a predictor. The weaker predictor, Y, should be ignored only when its relative Sharpe ratio is lower than its diversification benefits.

Our empirical work shows that the Sharpe ratio is the main determinant of the predictor redundancy problem of (1). In long–short portfolios, the Sharpe ratios of the within and across components differ substantially, and there is a high chance that the ratio in (1) becomes small. Therefore, an investor can improve the risk-return profile of the long–short portfolio by exclusively using the stronger predictor. In contrast, the difference in Sharpe ratios of the across and within components of long factors is small. In this case, the inequality rarely holds because the left-hand side is usually close to one. Therefore, the long investor is better off using the as-is signal that contains both components.

With the aggregate values of Sharpe ratios and correlation coefficients, we predict that using the across signal in addition to the within signal increases the Sharpe ratio of long–short and long-only factors with probabilities of 29% and 78%, respectively. Historical bootstraps of factor data confirm that our analytical probabilities are reasonably accurate: keeping the sector component produces better long–short factors in only 20% of the trials, while doing so produces better long-only factors in 78% of the trials.

Figure 1 shows the summary of our empirical results, specifically the difference between the Sharpe ratios of two factors: SR^* is the Sharpe ratio of the factor constructed using only the within-signal (i.e., sector-neutralized factor), and SR is the Sharpe ratio of the standard factor that sorts on the original signal, which consists of both across and within components. We use bootstraps to compute 1,000 differences in Sharpe ratios for the average factor of each construction method. A positive difference indicates that, on average, the sector-neutral factor dominates the original factor. The figure shows that the average change is positive for long–short factors regardless of the construction method. Thus, the long–short investor benefits from sector neutralizing. In contrast, average changes are negative for long-only factors. The largest drop from sector neutralizing happens for the value-weighted long-only factors that trade large stocks, which is arguably the most investable portfolio depicted in Figure 1.

The paper proceeds as follows. The second section relates the problem of redundancy or benefits of a predictor to the mean-variance theory of Markowitz (1952). The third section presents the empirical results. The final section offers some concluding remarks.

2 Analytics of sector bets

2.1 A mean-variance condition for redundancy of a predictor

A tech firm may have a high book-to-market ratio (BM) relative to other tech firms, but a low BM relative to non-tech firm. Although the firm would be considered a value company compared to other tech firms, a long–short sort on BM will short this firm because firms in the tech sector generally have a lower BM. In this context, the predictive power of the market-wide BM

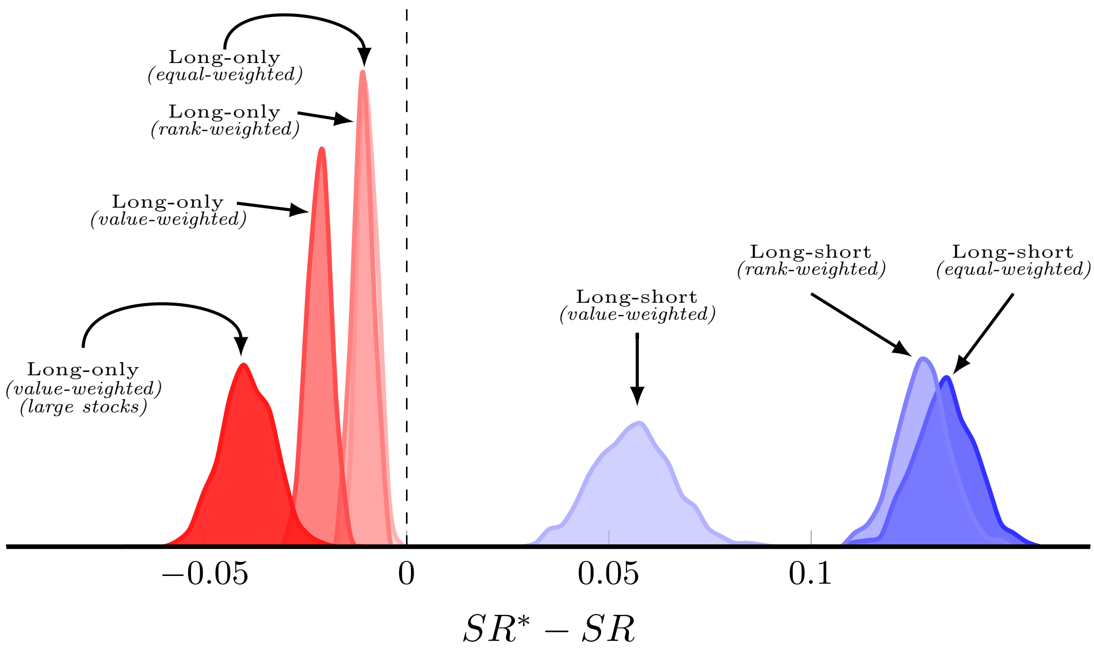


Figure 1: Impact of removing sector exposures on Sharpe ratios of equity factors.
We form two versions of each of the size, value, profitability, investment, and momentum factors. The first version sorts stocks based on firm characteristics; the second version sorts stocks based on firm characteristics minus their industry average. We compute the Sharpe ratio of the factors resulting from each version, and compute their difference. The factors are long–short or long-only, and the weighting schedules are equal, rank, or value. The number of sectors are 5, 10, 12, 17, 30, 38, 48 or 49 based on the industry classifications of Fama and French. The data is from 1963 to 2020. We compute the distributions by bootstrapping the factor return data by month.

results because sector BMs predict the cross-section of sector returns, firm-specific BMs predict the cross-section of firm-specific returns, or a combination of both. By extension, the return to a portfolio sorted on BM stems from its 1) sector exposure and 2) sector-neutralized (firm-specific) component. We follow the literature and use the terms *across* and *within* to refer to the sector and the sector-neutralized components, respectively. The return to a sort on a characteristic can be decomposed as

$$r_{factor} = r_{within} + r_{across}.$$

If the within component of BM predicts returns better than the across component, trading based on the within component will be more profitable than trading based on the across component, and we will have $SR_{within} > SR_{across}$, where SR denotes the Sharpe ratio. The question becomes, Should a mean-variance investor use the original signal, which has both components, or invest *only* using the component that predicts returns more accurately? Notice that because this is a static problem, we can view the portfolio that results from trading based on a predictive signal, such as sector BMs, as an asset whose existence is contingent on trading the signal. The answer to the problem of using or neglecting the signal should be settled by studying the properties of the eventual asset.

Our mean-variance investor seeks to maximize the overall Sharpe ratio of her portfolio, which has the well-known solution of $\frac{\Sigma^{-1}\mu}{1'\Sigma^{-1}\mu}$. In the case of the two assets generated by the within and across signals, denoted by subscripts w and a, respectively, the solution to optimal allocation to the across asset is

$$optimal\,weight(across\,component) = \frac{\mu_a \sigma_w^2 - \mu_w \,\mathrm{COV}(r_w, r_a)}{\mu_a \sigma_w^2 + \mu_w \sigma_a^2 + (\mu_w + \mu_a)\,\mathrm{COV}(r_w, r_a)}, \tag{2}$$

where μ and σ are mean and standard deviation, respectively, and $\mathrm{COV}(r_w, r_a)$ is the covariance between returns. An optimal weight of zero means that the Sharpe ratio-maximizing strategy is to invest in the within component exclusively and earn the Sharpe ratio of the within component. When the optimal weight is negative, the Sharpe ratio-maximizing strategy would be to invest in the within component and *short* the across component. Therefore, when equation (2) is non positive, the sector component is at best redundant and the mean-variance investor benefits from sector neutralizing the factor.

When the variance of the across and within components are equal, the solution in (2) sim-

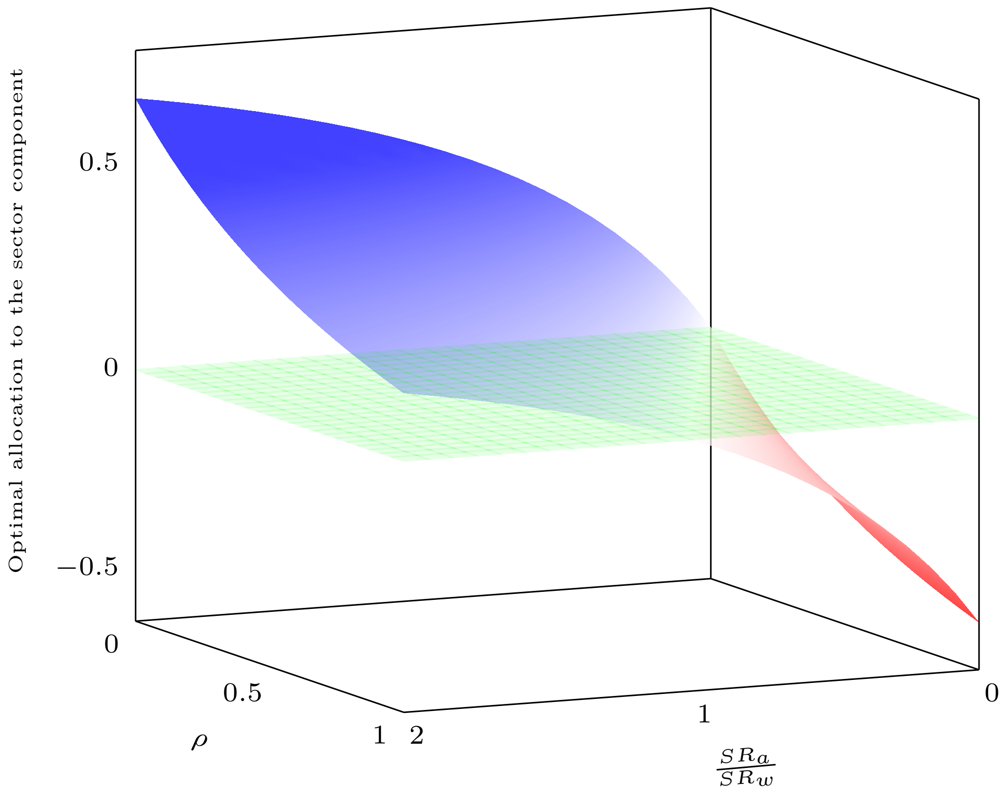


Figure 2: Optimal allocation to the sector component. This figure shows the optimal allocation to the sector component as a function of its Sharpe ratio relative to that of the within component, and the correlation between the two components. The area where optimal allocation to the sector bet is positive (negative) is shown using blue (red). The green plane displays an allocation of zero.

plifies to

$$optimal\,weight(across\,component) = \frac{\frac{SR_a}{SR_w} - \rho}{(1 + \frac{SR_a}{SR_w})(\rho + 1)}, \tag{3}$$

where ρ is the correlation coefficient between the two component returns. We plot this solution in Figure 3 by varying the correlation coefficient between 0 and 1, and the relative Sharpe ratios between 0 and 2. The figure shows that positive or negative optimal allocation to the sector bet are both possible outcomes. Reducing the Sharpe ratio of the sector component or increasing its correlation with the within component pushes the optimal allocation in deep red (negative allocation) and changing the parameters in the opposite direction lifts it to solid blue (positive allocation).

The optimal allocation in (3), which is an approximation of (2), assumes similar volatilities for the sector and within components. With positive correlations, however, the sign of the solution in (2) has an exact solution. The optimal weight in (2) is non positive when[1]

$$\frac{SR_a}{SR_w} \leq \rho. \tag{4}$$

The inequality in (4) is a trade-off between the efficiency and diversification benefits of the across component. If the across component is not correlated with the within component ($\rho = 0$), the across component brings large diversification benefits and will enter the portfolio even if it is associated with a small Sharpe ratio. But if the two components are highly correlated ($\rho \approx 1$), the across component can only enter the portfolio if it has a Sharpe ratio as high as the within component. In summary, the across component would have to have a low Sharpe ratio, a high correlation with the within component, or both, to be redundant.

The inequality allows us to discipline the across-versus-within problem. If the ratio of the two components Sharpe ratios is less than their correlation coefficient, the investor benefits from investing in the within component only. Otherwise, using the standard factor, which is similar to investing in both components, is the most mean-variance efficient.[2]

[1]The denominator in (2) is positive because means and correlations are positive. The sign of the numerator determines the sign of the identity:

$$\begin{aligned} optimal\ weight(a) \leq 0 \qquad \Leftrightarrow \qquad \mu_a\sigma_w^2 - \mu_w\,\mathrm{COV}(r_w, r_a) &\leq 0 \\ \rightarrow \mu_a\sigma_w^2 &\leq \mu_w\,\rho\sigma_w\sigma_a \\ \rightarrow \frac{\mu_a}{\sigma_a}\frac{\sigma_w}{\mu_w} &\leq \rho \\ \rightarrow \frac{SR_a}{SR_w} &\leq \rho \end{aligned}$$

In retrospect, we know the values for SR and ρ from the data, but in practice we should use prospective measures of SR and ρ. This difference, however, does not alter the thrust of the analysis here: the real-time investor should form a sector-neutral factor if the ratio of *expected* Sharpe ratios is less than the *expected* correlation between the two portfolios.

[2]We emphasize that our analysis is restricted to mean-variance efficiency. An investor that considers higher moments, for example a mean-variance-skewness investor, may not use (4) for decision making.

2.2 Decomposing signals and their returns

We analytically decompose the signals and returns. Following Ehsani, Hunstad, and Mehta (2020), denote the signals by C and returns by r, and index sectors and stocks by subscripts s and n, respectively. The average characteristic value of sector s and the average return of sector s are

$$C_s = \frac{1}{N}\sum_{n=1}^{N} C_{n,s} \quad \text{and} \quad r_s = \frac{1}{N}\sum_{n=1}^{N} r_{n,s}, \tag{5}$$

where S is the number of sectors and N is the number of stocks within each sector, and $C_{n,s}$ and $r_{n,s}$ are the characteristic and return of stock s in sector n, respectively. The return to the standard factor is

$$r_{factor} = \frac{1}{S \times N}\sum_{s}\sum_{n}\left(C_{n,s} - \overline{C}\right) r_{n,s}, \tag{6}$$

where $r_{n,s}$ is the return to stock n in sector s, and $\overline{C}$ is the average of the characteristics in the cross-section. The average characteristic score, C, in the cross-section equals the average scores of the sector characteristics as follows

$$\overline{C} = \frac{1}{S \times N}\sum_{s}\sum_{n} C_{n,s} = \frac{1}{S}\sum_{s=1}^{S} C_s. \tag{7}$$

The sector-neutralized factor invests in each stock based on its characteristic relative to the average characteristic of sector s. The return to the factor is

$$r_{within} = \frac{1}{S \times N}\sum_{s}\sum_{n}\left(C_{s,n} - C_s\right) r_{s,n}, \tag{8}$$

where C_s is the average characteristic score of sector s. We decompose factor returns as follows

$$r_{factor} = \frac{1}{S \times N} \sum_s \sum_n \left(C_{n,s} - \overline{C}\right) r_{n,s} = \frac{1}{S \times N} \sum_s \sum_n \left(C_{n,s} - \overline{C} + C_s - C_s\right) r_{n,s} \tag{9}$$

$$\rightarrow \underbrace{\frac{1}{S \times N} \sum_s \sum_n \left(C_{n,s} - \overline{C}\right) r_{n,s}}_{r_{factor}} = \underbrace{\frac{1}{S \times N} \sum_s \sum_n \left(C_{n,s} - C_s\right) r_{n,s}}_{r_{within\,(sector\text{-}neutralized)}} + \underbrace{\frac{1}{S \times N} \sum_s \sum_n \left(C_s - \overline{C}\right) r_{n,s}}_{r_{across}}. \tag{10}$$

3 Empirical results

We implement the decomposition in (10) for rank-weighted, equal-weighted, and value-weighted portfolios. These three methods and their compositions are the most popular approaches to portfolio construction in the literature and practice. Although the decomposition of (10) is motivated using the actual level of the characteristic, $C_{n,s}$, it can be applied to any weighting schedule. For example, for rank- or value-weighted portfolios, we just need to substitute $C_{n,s}$ with the normalized rank or market value of the stock; for equal-weighted portfolios, $C_{n,s}$ would be the reciprocal of the number of stocks in the portfolio.

3.1 An illustration of sector bets

Total exposure of a factor to sectors can be computed using the second term of the decomposition, $\frac{\sum_s \sum_n \left(C_s - \overline{C}\right)}{S \times N}$. We show the time series of sector exposures for the long–short value factor (the HML factor) in Figure 3. Over the 60-year period of our study, the standard value factor has consistently invested large amounts in Finance and Utilities and at the same time has shorted large amounts of Technology and Healthcare. Exposure to any risky asset, such as equity sectors, comes with volatility. The premium associated with this extra risk, and its covariance with

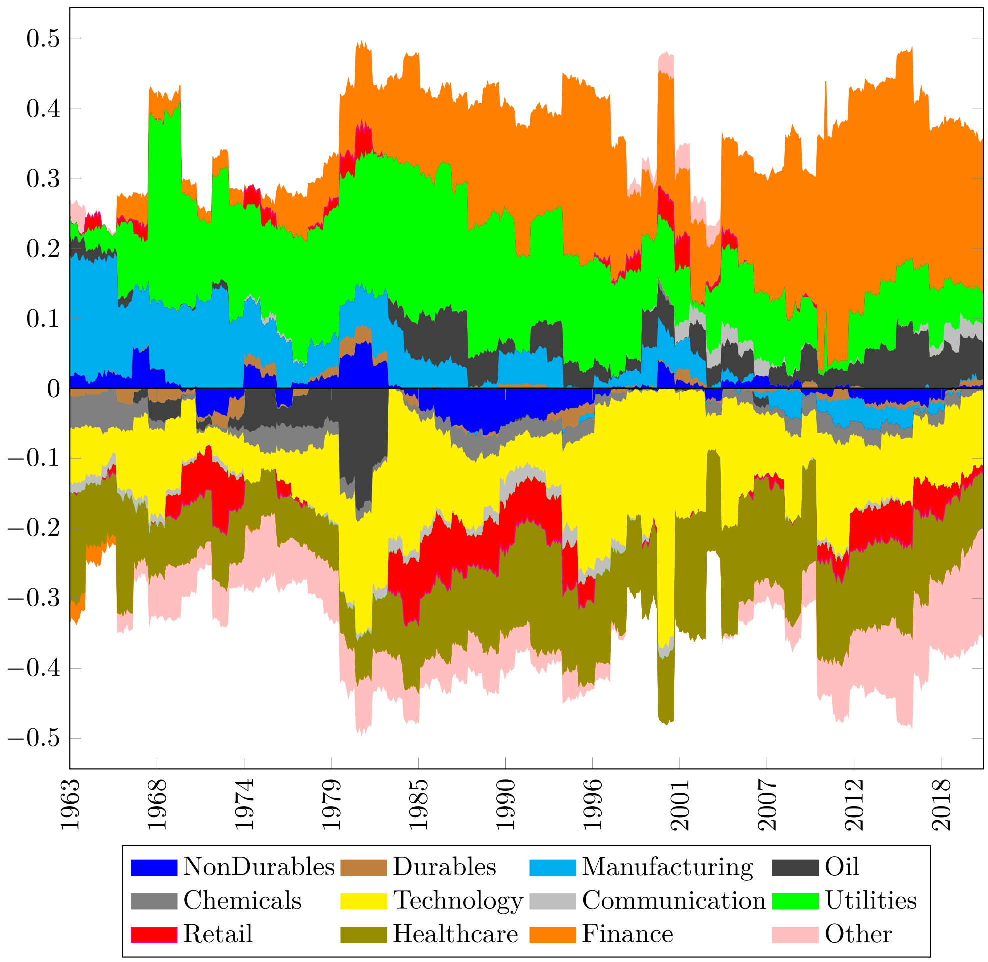


Figure 3: Sector bets of value. This figure shows the exposures of the long–short rank-weighted value factor over the 1963–2020 period to the 12 industry classifications of Fama and French.

the rest of the portfolio, determines its overall contribution to the factor's risk-return profile. We can construct the standard value factor with the sector exposures of Figure 3 or we can form a value factor using the within characteristics, such that the overall exposure to any one of the sectors is always zero. The sector-neutral value factor is likely less volatile than the standard value factor because it offsets the positive sector exposure of the long leg with an equal amount of negative exposure in the short leg.

3.2 Empirical return decomposition

We decompose the returns of equity factors using (10) at the intersection of the following factor construction techniques: 1) long–short, 2) long-only, and 1) equal weighted, 2) rank weighted, or 3) value weighted. We set the stage by reporting the summary of results for the two parameters of the condition in (4) for the average factor. Table 1 shows the results. Panel A shows the Sharpe ratios for the across and within components, their ratio, and the correlation between across and within components for the average factor. According to (4), the across component is redundant if the ratio of Sharpe ratios is less than the correlation coefficient. Panel A shows that for the average long–short factor, the ratio of the Sharpe ratios of the across component to the within component is 0.18, and the correlation between them is 0.46. Because 0.18 is less than 0.46, the long–short investor benefits from sorting on the within characteristic only.

The pattern in statistics of the average long-only factor is quite different. The Sharpe ratios of the within and across components are similar, with a ratio of 0.93. This means that the across component of the long factor is just slightly less mean-variance efficient than the within component. The average correlation between the returns of the across and within components is about 0.75. Because 0.94 is not less than 0.75, the inequality will not hold true and the optimal decision would be to use both signals. In summary, omitting the sector component is more likely to improve the long–short factor and degrade the long-only factor.[3]

[3]The long–short momentum seems to be an exception to this rule because strategies that trade momentum in the cross-section of industries earn significant returns Moskowitz and Grinblatt (1999). The Sharpe ratio of the sector component of momentum is between 64% (equal weighted) and 73% (value weighted) of the within component while the correlation between the two components is between 61% and 68%.

Table 1: Sharpe ratios and the correlation coefficient

The table shows Sharpe ratios of the across and within components of factors, the ratio of the Sharpe ratios (across to within), and their correlation coefficient. The factors are constructed using the Fama and French (2015) methodology. At the end of every June, we use accounting data for the fiscal year of the previous year to form 2×3 portfolios sorted on size and characteristic. Size breakpoint is the NYSE median market capitalization and the characteristic breakpoints are the NYSE 30th and 70th percentiles. We compute two high-minus-low returns for small and big stocks, and compute the long–short factor return as the average of the two. The long-only factor is constructed by averaging the returns of the high-large and high-small portfolios. The within and across components are obtained from equation (10). Sectors are defined using the Fama and French 12-sector classification. Data are monthly from 07/1963 to 12/2020 (690 months).

Panel A: Average of all factors-construction methodologies

	Long–short factors					Long-only factors			
	SR_{Across}	SR_{Within}	Ratio	ρ		SR_{Across}	SR_{Within}	Ratio	ρ
	0.10	0.53	0.18	0.46		0.50	0.54	0.93	0.75

Panel B: Equal weighted

	Long-short factors					Long-only factors			
	SR_{Across}	SR_{Within}	Ratio	ρ		SR_{Across}	SR_{Within}	Ratio	ρ
Size	-0.29	0.30	-0.97	0.18	Size	0.45	0.47	0.95	0.89
Value	0.05	0.82	0.06	0.64	Value	0.61	0.64	0.97	0.69
Profitability	0.15	0.24	0.64	0.64	Profitability	0.52	0.53	0.98	0.87
Investment	0.12	0.97	0.13	0.48	Investment	0.46	0.61	0.75	0.67
Momentum	0.37	0.57	0.64	0.68	Momentum	0.39	0.53	0.73	0.61

Panel C: Rank weighted

	Long-short factors					Long-only factors			
	SR_{Across}	SR_{Within}	Ratio	ρ		SR_{Across}	SR_{Within}	Ratio	ρ
Size	-0.26	0.24	-1.07	0.23	Size	0.43	0.42	1.03	0.90
Value	0.03	0.86	0.04	0.65	Value	0.60	0.65	0.93	0.70
Profitability	0.09	0.27	0.35	0.70	Profitability	0.52	0.53	0.98	0.85
Investment	0.21	1.05	0.20	0.50	Investment	0.46	0.59	0.78	0.68
Momentum	0.41	0.57	0.72	0.67	Momentum	0.53	0.58	0.91	0.73

Panel D: Value weighted

	Long-short factors					Long-only factors			
	SR_{Across}	SR_{Within}	Ratio	ρ		SR_{Across}	SR_{Within}	Ratio	ρ
Size	-0.27	0.26	-1.03	-0.17	Size	0.47	0.42	1.12	0.87
Value	0.01	0.46	0.03	0.43	Value	0.57	0.51	1.11	0.71
Profitability	0.25	0.46	0.54	0.31	Profitability	0.56	0.53	1.05	0.83
Investment	0.15	0.41	0.37	0.31	Investment	0.44	0.54	0.80	0.59
Momentum	0.39	0.53	0.73	0.61	Momentum	0.53	0.55	0.96	0.65

We next present the factor-by-factor results for the case of the 12-sector classification. Table 2 shows the factor-by-factor decomposition results.[4] The columns on the left of Table 2 show the returns to each long–short factor, the return to its across component, and the return to its within component. Our focus is on the Sharpe ratios (t-values) of the factors and components. If the t-value of the within component is larger than that of the factor, the mean-variance investor is better off trading the within component only. Indeed, consistent with Asness, Porter, and Stevens (2000) and Ehsani, Hunstad, and Mehta (2020), we find that the within component of almost every long–short factor earns a larger t-ratio than the factor. Size, value, and investment factors encounter the largest gains.

The columns on the right in Table 2 show the results for long-only factors. The trend in t-values of long factors is the opposite of the long–short factors. Here, the as-is factor always earns the highest t-value. It is important to emphasize that even for long-only factors the within component is still more mean-variance efficient than the across component. Consider the long-only momentum as an illustration. Regardless of the weighting schedule, the within component of momentum always dominates the across component. For equal-weighted momentum, for example, the within component earns a t-value of 4.60, but the across component earns a t-value of 3.95. Yet, the t-value of as-is momentum—that is, the sum of within and across momentums—is the largest, indicating that the long-only investor should *not* seek to invest exclusively in the stronger component. This result may seem confusing at first glance, but it is completely explained using the condition in (4): the across component is redundant only when its Sharpe ratio is sufficiently low. Neither of the across components of long-only factors meets this low Sharpe-ratio bar. Therefore, the long factor investor is better off investing in the factor

[4] We examine the robustness of results by running the decomposition on all eight Fama and French industry classifications in section 3.4.

that sorts on the characteristic as it stands.

Table 2: Factor returns

The table shows mean returns and t-values to factor returns and their within and across components using the decomposition of equation (10). We use the 12-sector classification of Fama and French. Data are monthly from 07/1963 to 12/2020 (690 months).

Panel A: Equal weighted

	Long–short factors				Long-only factors		
	Factor	Across	Within		Factor	Across	Within
Size	0.23	-0.06	0.28	Size	0.92	0.08	0.84
	(1.72)	(-2.23)	(2.29)		(3.58)	(3.38)	(3.57)
Value	0.41	0.02	0.38	Value	1.01	0.19	0.82
	(3.65)	(0.38)	(6.24)		(5.03)	(4.65)	(4.82)
Profitability	0.15	0.04	0.11	Profitability	0.88	0.10	0.77
	(1.71)	(1.16)	(1.79)		(4.09)	(3.95)	(4.04)
Investment	0.40	0.02	0.38	Investment	1.00	0.07	0.93
	(5.96)	(0.93)	(7.34)		(4.64)	(3.46)	(4.62)
Momentum	0.58	0.14	0.44	Momentum	1.05	0.16	0.89
	(4.14)	(2.80)	(4.36)		(4.68)	(3.95)	(4.60)

Panel B: Rank weighted

	Long-short factors				Long-only factors		
	Factor	Across	Within		Factor	Across	Within
Size	0.19	-0.05	0.25	Size	0.88	0.09	0.80
	(1.33)	(-1.94)	(1.82)		(3.23)	(3.27)	(3.19)
Value	0.52	0.02	0.50	Value	1.06	0.20	0.86
	(3.90)	(0.25)	(6.52)		(5.08)	(4.55)	(4.90)
Profitability	0.20	0.03	0.17	Profitability	0.89	0.11	0.78
	(1.71)	(0.71)	(2.04)		(4.04)	(3.91)	(3.98)
Investment	0.56	0.04	0.52	Investment	1.04	0.07	0.97
	(6.80)	(1.56)	(7.96)		(4.49)	(3.47)	(4.46)
Momentum	0.72	0.18	0.55	Momentum	1.11	0.18	0.93
	(4.23)	(3.07)	(4.29)		(4.49)	(3.99)	(4.38)

Panel C: Value weighted

	Long-short factors				Long-only factors		
	Factor	Across	Within		Factor	Across	Within
Size	0.19	-0.06	0.25	Size	0.79	0.09	0.70
	(1.51)	(-2.02)	(1.97)		(3.26)	(3.56)	(3.18)
Value	0.23	0.01	0.22	Value	0.82	0.20	0.61
	(2.07)	(0.09)	(3.52)		(4.22)	(4.33)	(3.90)
Profitability	0.30	0.07	0.23	Profitability	0.81	0.13	0.67
	(3.53)	(1.89)	(3.51)		(4.14)	(4.21)	(4.01)
Investment	0.21	0.04	0.16	Investment	0.81	0.07	0.73
	(2.82)	(1.17)	(3.13)		(4.19)	(3.30)	(4.11)
Momentum	0.59	0.18	0.41	Momentum	0.94	0.19	0.75
	(3.98)	(2.93)	(3.99)		(4.43)	(4.03)	(4.20)

3.3 Sector-level decompositions

Table 3 provides a detailed decomposition of the long–short value returns at the sector level. We present the average exposure to each sector and the resulting return and variance contributions, to the overall return and variance of the portfolio.[5] We report the results for the original factor in the top panel and for its two components in the following panels. By construction, exposures and returns of the original factor equal the sum of the exposures and returns of the sector and within components. The sum of the variances of the sector and within components does not add up to the variance of the original factor because of their covariance.

An interesting result is that the within returns of the last panel are positive for every sector. As a result, the total contribution of the within component is highly significant (0.22% with a *t*-value of 3.52). The net return emerging from the sector component, reported in the second panel, is close to zero. Trading BM within the cross-section of every sector is highly profitable, while trading BM in the cross-section of sectors is not. This observation suggests that the entire predictive power of market-wide BM stems from the information in its firm-specific component. A long–short investor that sorts stocks based on a raw BM signal contaminates the useful within-sector information by the noise of the across-sector component. Once again consider the example of tech firms. The original BM shorts tech because most tech companies are considered to be growth companies when compared to the average company. Therefore, the standard value factor does not exploit the variation of BM *within* the Tech sector because tech stocks are all sent to the short leg of the portfolio. Table 3 shows that, however, BM is priced in the cross-section of Tech companies with a premium of 0.04% (*t*-value of 2.07). This result explains our

[5] We compute variance contributions by computing $1_{1\times12}\Sigma_{12\times12}$. That is, each sector's contribution is the sum of the elements of the associated column of the 12×12 covariance matrix of 12 sector returns. For example, if Durables is the second sector, then the sum of the elements in the second column gives the variance contribution of Durables to the portfolio.

earlier findings: the long–short factor sorted on the raw characteristic is less efficient than an alternative sort on the within component, because the raw characteristic is contaminated with the noise from the across component.

Tables 4 and 5 show sector-level decompositions for a long-only and a short-only value factor. Table 4 shows that, as expected, the standard long value factor has positive sector exposures to all sectors, and as a result, each sector contributes positively to its total return of 0.82% (t-value of 4.22). The second panel of Table 4 shows the contribution of each sector to the sector component of long value.[6] The sector component earns a large t-ratio of 4.33 that is larger than the t-ratio of 3.90 earned by the within component shown in the third panel. In fact, the Sharpe-ratio maximizing optimal mix of the sector and within components would be to put 86% in the sector component and 14% in the within component; this long-only "factor" would have earned a t-value of 4.50. The main takeaway from Table 4 is that both across and within components contribute to returns of long-only factors.

Tables 5 shows the same results for a short value factor. The standard short value factor is a short position in stocks with a BM below the average BM. This portfolio loses an average of −0.59% per month with a t-value of −2.75. The following two panels decompose this premium. The sector component generates −0.20% of this premium (t-value of −2.72) and the within component generates −0.39% (t-value of −2.57). Notice that the −0.20% contribution of the sector component is exactly equal to the 0.20% of the sector component of Table 4. The re-

[6]The average exposure to some sectors is negative. How can a long value factor have negative exposure to certain sectors? Note that we compute sector exposures by finding $\frac{1}{S\times N}\sum_{s}\sum_{n}\left(C_s - \overline{C}\right)$. Suppose the average BM in the entire cross-section is 1. Also assume that we want to form the value factor using four stocks of a sector, and that these stocks have BMs of 3, 2, −2, and −3. The mean BM of this sector is 0. The long value factor invests in the first two stocks only; it allocates $(3-1)/2 = 1$ to the first stock and $(2-1)/2 = 0.5$ to the second. Now lets decompose these weights into within and across components. The within strategy sorts using the difference with the sector mean instead of the cross-sectional mean, so it allocates $(3-0)/2 = 1.5$ to the first stock and $(2-0)/2 = 1$ to the second. This means that in long-only constructions, the within component of sectors whose BMs are smaller than the average BM will be positive, while the sector exposures of the sector component will be negative. This example explains the negative exposures in the second panel of Tables 4.

turn from the within component, however, is -0.39% and smaller than the 0.61% of the within component of the long leg.

A comparison of the results of Tables 4 and 5 indicates that the predictive power of BM for the long leg is not through firm-specific BMs; the trader can do just as well by trading sector BMs. In contrast, the predictive power of BM for the short leg is through the within or firm-specific channel. To summarize, the within component of BM is strong in identifying the underperforming stocks of each sector that should go in the short leg, but does not predict the outperforming stocks of the long leg. That is, the within long–short value outperforms the standard long–short value factor only because the within component is better at forming the short leg.

We next study the volatility of the components. The variance of the sector component of the long value factor (Table 4) is 1.53 and that of the sector component of the short value factor (Table 5) is 3.59. The large variance of the sector bet of the short leg therefore cannot be properly diversified by the variance of the long leg in the long–short portfolio. The variance assymetry is reflected in the large variance of 3.20 (Table 3) for sector component of the long–short value factor. At the same time, the variance assymetry between the within components is much smaller: the within component of long value has a variance of 17.74 and the within component of short value has a similar variance of 16.01.

To summarize, the market-wide predictive power of a characteristic can arise because it predicts the cross-section of sectors or because it predicts the cross-section of the elements within each sector. The results of this section show that the across BM is better at predicting the outperforming *sectors* and the within BM is better at predicting the underperforming *stocks* within each sector. In retrospect, the maximum Sharpe-ratio strategy of the long–short value factor would be to form the long leg using sector BMs and to form the short leg using stock-specific

BMs.

Table 3: Exposure and return contribution by each sector to the long–short value factor

The table shows the average sector exposure and the resulting variance and returns from these exposures by each sector for the long–short value-weighted value factor. We report the sector contributions for the overall factor and for its two components. Data are monthly from 07/1963 to 12/2020 (690 months).

	NonDur	Dur	Manufac	Oil	Chemical	Tech	Comm	Util	Retail	Health	Finance	Other	Total
Factor													
Average	-0.02	-0.01	0.04	0.03	-0.03	-0.15	0.03	0.12	-0.04	-0.11	0.17	-0.02	0.00
Exposure	(-2.87)	(-1.86)	(2.30)	(2.95)	(-9.74)	(-15.28)	(2.37)	(8.06)	(-6.79)	(-13.04)	(6.85)	(-1.98)	
Average	0.00	0.01	0.08	0.06	-0.01	-0.07	0.01	0.09	-0.01	-0.09	0.18	-0.01	0.23
Return	(-0.38)	(1.19)	(3.00)	(2.53)	(-1.25)	(-1.38)	(0.49)	(4.35)	(-0.51)	(-2.91)	(3.28)	(-0.36)	(2.07)
Variance Contribution	0.32	0.22	0.58	0.49	0.15	2.62	0.28	0.41	0.51	1.03	1.14	0.88	8.64
Sector component													
Average	-0.02	-0.01	0.04	0.03	-0.03	-0.15	0.03	0.12	-0.04	-0.11	0.17	-0.02	0.00
Exposure	(-2.87)	(-1.86)	(2.30)	(2.95)	(-9.74)	(-15.28)	(2.37)	(8.06)	(-6.79)	(-13.04)	(6.85)	(-1.98)	
Average	-0.01	0.00	0.04	0.02	-0.02	-0.11	0.00	0.09	-0.01	-0.12	0.16	-0.03	0.01
Return	(-1.58)	(0.56)	(2.04)	(0.80)	(-3.78)	(-2.09)	(-0.24)	(4.70)	(-1.42)	(-4.00)	(3.49)	(-2.01)	(0.09)
Variance Contribution	0.05	0.03	-0.01	0.16	0.05	1.86	0.05	0.18	0.11	0.82	-0.08	-0.01	3.20
Within component													
Average	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Exposure	.	.	.	.	.	.	.	.	.	.	.	.	
Average	0.01	0.01	0.03	0.04	0.01	0.04	0.01	0.00	0.01	0.04	0.02	0.02	0.22
Return	(1.46)	(0.95)	(1.97)	(4.37)	(2.77)	(2.07)	(0.85)	(-0.19)	(0.70)	(3.16)	(0.98)	(1.34)	(3.52)
Variance Contribution	0.13	0.15	0.43	0.12	0.10	0.41	0.21	0.05	0.26	0.20	0.38	0.50	2.94

Table 4: Exposure and return contribution by each sector to the long-only value factor

The table shows the average sector exposure and the resulting variance and returns from these exposures by each sector for the long-only value-weighted value factor. We report the sector contributions for the overall factor and for its two components. Data are monthly from 07/1963 to 12/2020 (690 months).

	Factor												
	NonDur	Dur	Manufac	Oil	Chemical	Tech	Comm	Util	Retail	Health	Finance	Other	Total
Average Exposure	0.06 (10.36)	0.04 (8.64)	0.15 (7.05)	0.07 (10.87)	0.02 (9.86)	0.05 (10.29)	0.06 (5.98)	0.14 (10.56)	0.07 (19.80)	0.02 (9.39)	0.23 (9.31)	0.11 (18.06)	1.00
Average Return	0.05 (3.25)	0.03 (2.50)	0.12 (2.78)	0.06 (2.65)	0.02 (2.94)	0.06 (3.38)	0.02 (1.26)	0.09 (4.10)	0.06 (3.16)	0.02 (3.82)	0.20 (3.34)	0.10 (3.38)	0.82 (4.22)
Variance Contribution	1.56	1.24	4.28	1.89	0.57	1.64	0.96	1.41	2.11	0.46	6.85	3.56	26.53
	Sector component												
	NonDur	Dur	Manufac	Oil	Chemical	Tech	Comm	Util	Retail	Health	Finance	Other	Total
Average Exposure	0.01 (2.36)	0.00 (-0.52)	0.03 (3.02)	0.03 (4.77)	0.00 (-6.25)	-0.03 (-16.94)	0.02 (2.92)	0.12 (8.70)	-0.01 (-2.89)	-0.01 (-10.06)	0.13 (6.35)	-0.01 (-2.44)	0.26
Average Return	0.01 (1.79)	0.00 (0.64)	0.04 (3.15)	0.01 (0.97)	-0.01 (-3.21)	-0.04 (-4.72)	0.00 (-0.09)	0.09 (4.61)	0.00 (0.80)	-0.03 (-5.04)	0.12 (3.49)	-0.01 (-2.13)	0.20 (4.33)
Variance Contribution	0.02	0.00	0.09	0.26	-0.03	-0.10	0.06	0.41	-0.01	-0.04	0.95	-0.09	1.53
	Within component												
	NonDur	Dur	Manufac	Oil	Chemical	Tech	Comm	Util	Retail	Health	Finance	Other	Total
Average Exposure	0.06 (18.45)	0.04 (12.24)	0.12 (12.73)	0.05 (13.92)	0.02 (13.64)	0.07 (17.26)	0.04 (11.02)	0.02 (10.56)	0.07 (43.30)	0.03 (13.32)	0.10 (19.09)	0.12 (23.47)	0.75
Average Return	0.04 (3.28)	0.03 (2.44)	0.08 (2.37)	0.05 (3.31)	0.02 (3.25)	0.10 (3.99)	0.02 (1.73)	0.00 (0.17)	0.05 (2.98)	0.05 (4.79)	0.08 (2.69)	0.11 (3.45)	0.61 (3.90)
Variance Contribution	1.08	1.02	2.97	0.87	0.61	2.19	0.63	0.12	1.80	0.73	2.54	3.20	17.74

Table 5: Exposure and return contribution by each sector to the short-only value factor

The table shows the average sector exposure and the resulting variance and returns from exposure to each sector for the short-only value-weighted value factor. We report the sector contributions for the overall factor and for its two components. Data are monthly from 07/1963 to 12/2020 (690 months).

	Overall factor												
	NonDur	Dur	Manufac	Oil	Chemical	Tech	Comm	Util	Retail	Health	Finance	Other	Total
Average Exposure	-0.08	-0.05	-0.12	-0.04	-0.05	-0.20	-0.03	-0.01	-0.10	-0.13	-0.06	-0.13	-1.00
	(-16.25)	(-9.64)	(-15.32)	(-6.44)	(-15.57)	(-18.77)	(-9.52)	(-2.95)	(-28.72)	(-15.06)	(-12.91)	(-12.81)	
Average Return	-0.05	-0.02	-0.04	-0.01	-0.02	-0.13	-0.02	0.00	-0.06	-0.10	-0.03	-0.10	-0.59
	(-3.26)	(-1.90)	(-1.44)	(-0.28)	(-2.55)	(-2.11)	(-1.89)	(-0.30)	(-2.71)	(-3.29)	(-2.09)	(-2.73)	(-2.75)
Variance Contribution	1.86	1.28	3.65	1.35	1.10	8.12	0.97	0.15	3.01	3.58	1.55	4.84	31.47

	Sector component												
	NonDur	Dur	Manufac	Oil	Chemical	Tech	Comm	Util	Retail	Health	Finance	Other	Total
Average Exposure	-0.03	-0.01	0.01	0.00	-0.03	-0.13	0.01	0.01	-0.03	-0.10	0.05	-0.01	-0.27
	(-4.98)	(-3.18)	(1.16)	(0.13)	(-12.24)	(-13.30)	(1.17)	(1.30)	(-8.08)	(-17.15)	(7.85)	(-1.68)	
Average Return	-0.02	0.00	0.00	0.00	-0.02	-0.07	0.00	0.00	-0.02	-0.09	0.04	-0.02	-0.20
	(-3.24)	(0.17)	(0.26)	(0.18)	(-3.05)	(-1.58)	(-0.20)	(0.42)	(-2.14)	(-3.46)	(3.00)	(-1.76)	(-2.72)
Variance Contribution	0.16	0.05	-0.01	0.14	0.16	2.12	-0.02	-0.08	0.24	1.07	-0.38	0.14	3.59

	Within component												
	NonDur	Dur	Manufac	Oil	Chemical	Tech	Comm	Util	Retail	Health	Finance	Other	Total
Average Exposure	-0.06	-0.04	-0.12	-0.05	-0.02	-0.07	-0.04	-0.02	-0.07	-0.02	-0.10	-0.12	-0.74
	(-18.45)	(-11.09)	(-12.73)	(-13.69)	(-12.53)	(-16.51)	(-12.20)	(-11.16)	(-43.30)	(-10.75)	(-18.37)	(-23.47)	
Average Return	-0.03	-0.02	-0.04	-0.01	-0.01	-0.06	-0.02	0.00	-0.05	-0.01	-0.06	-0.09	-0.39
	(-2.39)	(-2.03)	(-1.30)	(-0.58)	(-1.37)	(-2.34)	(-1.30)	(-0.51)	(-2.66)	(-1.06)	(-2.74)	(-2.58)	(-2.57)
Variance Contribution	1.03	0.91	3.01	0.76	0.39	1.68	0.82	0.32	1.63	0.31	1.99	3.16	16.01

3.4 Sensitivity to the choice of sector classification

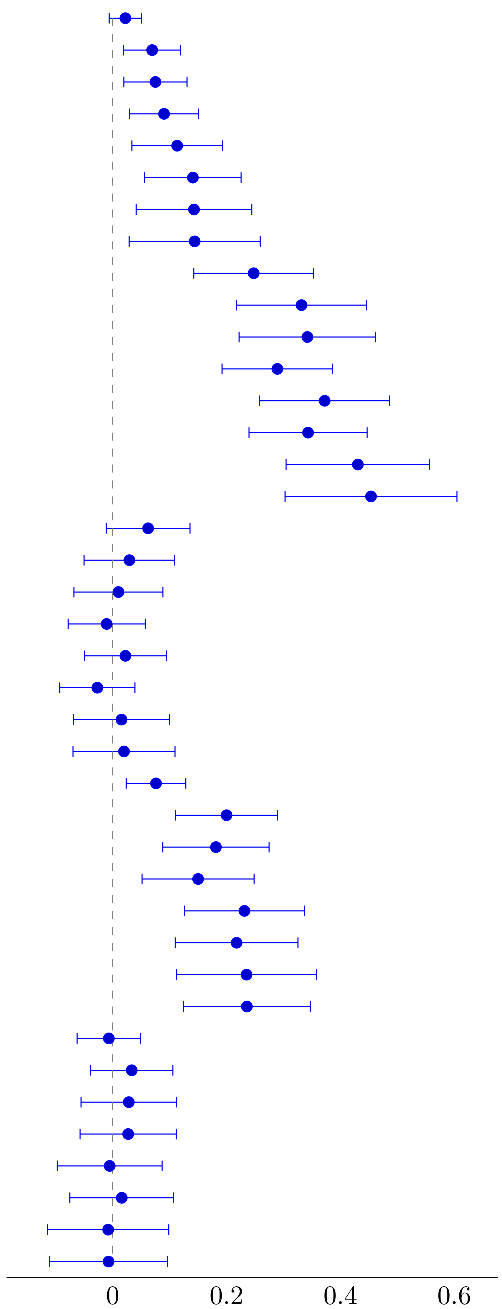
In the interest of brevity and to conserve space, the previous results were based on the 12 sector classifications. This section tests the sensitivity of the inferences to the choice of sector classification. We present the results for two construction methods most impacted by hedging out sector bets. The equal-weighted long–short portfolios benefit the most and the long-only value-weighted deteriorate the most. The Sharpe ratio changes of other construction techniques fall somewhere between these two.

Figure 4 shows differences between Sharpe ratios of sector-neutral and standard factors for the equal-weighted long–short construction approach. The figure shows that nearly all the factors constructed using this method benefit from sector neutrality regardless of the sector classification. The largest improvement of 0.45 units in annualized Sharpe ratio occurs for the value factor that hedges out exposure to 49 industries.

Figure 5 shows the changes in Sharpe ratios from removing sector bets for the value-weighted long-only factors. Once again, we find that the choice of sector classification is generally inconsequential. The Sharpe ratios of long-only factors drop if the sector component is neglected regardless of the classification. Comparing Figure 4 and Figure 5, we conclude that the key determinant for whether sector neutrality is beneficial is the choice of long–short versus long-only. The former group is more likely to benefit and the latter group is more likely to deteriorate.

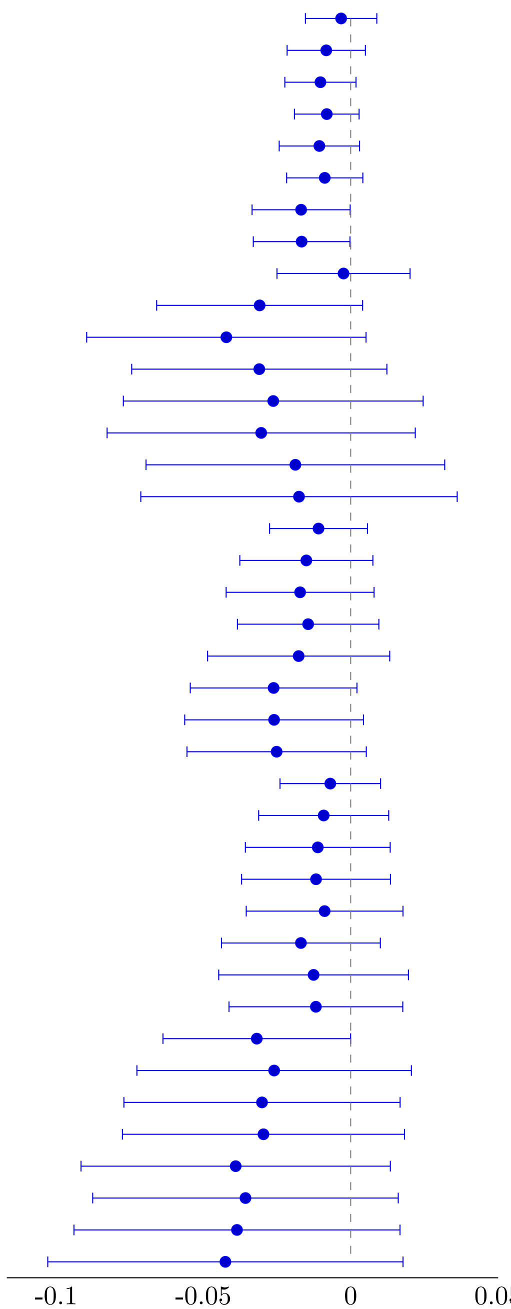
4 Conclusion

Investors that pursue factor investing face numerous choices - and the choices don't end once a set of factors is determined. For example, should the factors be sector neutralized or not? If sector neutralization is pursued, how many sectors? Is the decision contingent on the particular



Factor	$SR^* - SR$	95% CI
Size-05	0.02	(−0.01 to +0.05)
Size-10	0.07	(+0.02 to +0.12)
Size-12	0.08	(+0.02 to +0.13)
Size-17	0.09	(+0.03 to +0.15)
Size-30	0.11	(+0.04 to +0.19)
Size-38	0.14	(+0.06 to +0.23)
Size-48	0.14	(+0.04 to +0.24)
Size-49	0.14	(+0.05 to +0.26)
Value-05	0.25	(+0.15 to +0.35)
Value-10	0.33	(+0.23 to +0.45)
Value-12	0.34	(+0.23 to +0.46)
Value-17	0.29	(+0.18 to +0.39)
Value-30	0.37	(+0.26 to +0.49)
Value-38	0.34	(+0.24 to +0.45)
Value-48	0.43	(+0.30 to +0.56)
Value-49	0.45	(+0.32 to +0.60)
Profitability-05	0.06	(−0.02 to +0.14)
Profitability-10	0.03	(−0.05 to +0.11)
Profitability-12	0.01	(−0.07 to +0.09)
Profitability-17	−0.01	(−0.07 to +0.06)
Profitability-30	0.02	(−0.06 to +0.09)
Profitability-38	−0.03	(−0.10 to +0.04)
Profitability-48	0.02	(−0.07 to +0.10)
Profitability-49	0.02	(−0.07 to +0.11)
Investment-05	0.08	(+0.02 to +0.13)
Investment-10	0.20	(+0.12 to +0.29)
Investment-12	0.18	(+0.10 to +0.27)
Investment-17	0.15	(+0.06 to +0.25)
Investment-30	0.23	(+0.13 to +0.34)
Investment-38	0.22	(+0.12 to +0.33)
Investment-48	0.24	(+0.13 to +0.36)
Investment-49	0.24	(+0.12 to +0.35)
Momentum-05	−0.01	(−0.06 to +0.05)
Momentum-10	0.03	(−0.03 to +0.11)
Momentum-12	0.03	(−0.04 to +0.11)
Momentum-17	0.03	(−0.04 to +0.11)
Momentum-30	−0.01	(−0.08 to +0.09)
Momentum-38	0.02	(−0.05 to +0.11)
Momentum-48	−0.01	(−0.09 to +0.10)
Momentum-49	−0.01	(−0.09 to +0.10)

Figure 4: The impact of industry classification on changes in Sharpe ratios—long-short factors. This figure shows the changes in Sharpe ratios as a result of neutralizing sector exposures of equal-weighted long-short factor portfolios. For each factor, we compute the difference between the Sharpe ratio of its industry-neutral version and its standard version. We use all eight different industry classification of Fama and French. Value-12, for example, refers to the difference between the Sharpe ratio of the sector-neutral Value and the original Value using the 12 industry classification. We compute confidence interval for this difference in Sharpe ratios by bootstrapping the data by time.



Factor	SR* − SR	95% CI
Size-05	0.00	(−0.02 to +0.01)
Size-10	−0.01	(−0.02 to +0.01)
Size-12	−0.01	(−0.02 to +0.00)
Size-17	−0.01	(−0.02 to +0.00)
Size-30	−0.01	(−0.02 to +0.00)
Size-38	−0.01	(−0.02 to +0.00)
Size-48	−0.02	(−0.03 to +0.00)
Size-49	−0.02	(−0.03 to +0.00)
Value-05	0.00	(−0.02 to +0.02)
Value-10	−0.03	(−0.07 to +0.00)
Value-12	−0.04	(−0.09 to +0.01)
Value-17	−0.03	(−0.08 to +0.01)
Value-30	−0.03	(−0.07 to +0.02)
Value-38	−0.03	(−0.08 to +0.02)
Value-48	−0.02	(−0.07 to +0.03)
Value-49	−0.02	(−0.07 to +0.04)
Profitability-05	−0.01	(−0.03 to +0.01)
Profitability-10	−0.02	(−0.04 to +0.01)
Profitability-12	−0.02	(−0.04 to +0.01)
Profitability-17	−0.01	(−0.04 to +0.01)
Profitability-30	−0.02	(−0.05 to +0.01)
Profitability-38	−0.03	(−0.05 to +0.00)
Profitability-48	−0.03	(−0.06 to +0.00)
Profitability-49	−0.03	(−0.05 to +0.01)
Investment-05	−0.01	(−0.02 to +0.01)
Investment-10	−0.01	(−0.03 to +0.01)
Investment-12	−0.01	(−0.04 to +0.01)
Investment-17	−0.01	(−0.03 to +0.01)
Investment-30	−0.01	(−0.04 to +0.02)
Investment-38	−0.02	(−0.05 to +0.01)
Investment-48	−0.01	(−0.04 to +0.02)
Investment-49	−0.01	(−0.04 to +0.02)
Momentum-05	−0.03	(−0.06 to +0.00)
Momentum-10	−0.03	(−0.07 to +0.02)
Momentum-12	−0.03	(−0.08 to +0.02)
Momentum-17	−0.03	(−0.07 to +0.02)
Momentum-30	−0.04	(−0.09 to +0.01)
Momentum-38	−0.04	(−0.08 to +0.02)
Momentum-48	−0.04	(−0.09 to +0.02)
Momentum-49	−0.04	(−0.10 to +0.02)

Figure 5: The impact of industry classification on changes in Sharpe ratios—long-only factors. This figure shows the changes in Sharpe ratios of sector-neutral long-only factors for each of the eight industry classification of Fama and French. The rest of the figure is as in Figure 4.

factor? Are the choices potentially different for the long-only investor compared to the long-short investor? Our paper provides insights on each of these four questions.

While previous research has provided some evidence in favor of sector neutrality for certain factors, we present a simple framework with two sources of predictability: across sector and within sector. The question of whether to use both sources of predictability or to just focus on the within sector (i.e., sector neutralization) is related to the static two risky-asset mean variance problem. We derive the condition whereby it is optimal to sector neutralize. The condition relies on the Sharpe ratios of the individual sources of predictability as well as the correlation of the two sources.

We show in bootstrap simulations that our analytical framework does a good job of matching the data. While others have argued for the benefits of sector neutralization, our framework shows how much benefit to expect for various different factors - as well as the drivers. Our model also predicts that it is unlikely that sector neutralization is beneficial for the long-only investor and the empirical results are consistent with this prediction.

While our analytical model does not predict the number of sectors that should be used, our empirical results suggest a very small number of industries, such as five, is a mistake. The results also show diminishing contributions from large number of industries. In addition, the number of industries chosen is related to the particular factor.

Our analysis has two caveats. First, our empirical results based on the mean-variance framework are ex post. That is, sector neutralization of long–short (long-only) factors is generally beneficial (detrimental) based on an ex-post, historical analysis. Indeed, some previous research also comes to this conclusion. However, the importance of our paper is that we show "why". While the empirical analysis is ex post, our framework based on the Sharpe ratios of the factor within-industry and across-industry predictability as well as the correlation can be used on an ex ante basis. It is reasonable to expect that both Sharpe ratios and correlations can change through time. Our method gives investors a metric to track the expected contribution of sector neutralization in factor portfolios for active portfolio management.

The second caveat is the mean-variance framework itself. While commonplace in investment management, the choice of whether to neutralize or not assumes that investors only care about mean and variance. However, it is well known that investors prefer positively skewed returns and that most factor returns are not normally distributed. For example, suppose the sector component has positive skew. An investor might think twice about expunging sector exposures - even if our mean variance framework suggests neutralization. This is a topic of on-going research.

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A Estimating likelihoods using sample moments and historical simulation

The condition in equation (1) that relates the ratio of Sharpe ratios to the correlation between the two components decides whether an investor should take sector bets. We examine three ways to compute weights and eight ways to define sectors for each of the five factors (size, value, profitability, investment, and momentum), for a total of $3 \times 8 \times 5 = 120$ different long–short or long-only factors. Table A1 shows the average moments of the two parameters of (1) across these variations.

Table A1 shows that the mean correlation between the within and across components of long–short factors is 0.49. This figure implies that an investor should avoid the across component if she believes that the Sharpe ratio of the across component is less than half of the within component. The across component meets this low Sharpe-ratio bar by earning a monthly Sharpe ratio of 0.03, which is less than 20% of that of the within component.

Unsurprisingly, because all long portfolios have significant exposure to the market factor, the correlation between the components of long-only factors is substantially higher at 0.79. Because of this high correlation, the bar for the sector component is much higher: its Sharpe ratio should be at least 79% of the within component. It turns out that the across component of long factors meets this high bar by earning a monthly Sharpe ratio of 0.15, which is more than 90% of the Sharpe ratio of the within component (note that the bar was 80%). It is also important to note that these two moments—Sharpe ratio and correlation—are much less volatile in long portfolios. Correlations and Sharpe ratios of long portfolios fall in a narrow range because of the large exposure of long portfolios to the market; exposure to the market makes long portfolios very similar.

The moments in Table A1 estimate the likelihood that the factor portfolio benefits from neutralizing its sector bets. We must have $\rho \times SR_{within} > SR_{across}$ for the across component to be redundant. If the correlation parameter and Sharpe ratios of the within component do not co-vary, the product on the left side of the inequality will have a mean of $0.49 \times 0.16 = 0.08$ with a standard deviation of $\sqrt{0.23^2 \times 0.07^2 + 0.23^2 \times 0.16^2 + 0.07^2 \times 0.49^2} = 0.05$. According to Table A1, the parameter on the right-hand side of the inequality, SR_{across}, has a distribution of $\mathcal{N}(0.03, 0.07^2)$. The probability that a variable drawn from $\mathcal{N}(0.08, 0.05^2)$ is less than another *independent* variable drawn from $\mathcal{N}(0.03, 0.07^2)$ is 29%. That is, we expect a long–short factor to benefit from adding sector bets only about 29% of the time. This is an approximation because in data the right- and left-hand sides of the inequality are positively correlated. A similar back-of-the-envelope calculation using the moments of the long-only portfolios estimates a 78% probability that adding sector bets increases the Sharpe ratio.

We now turn to historical data to estimate these likelihoods and compare them with their analytical counterparts. We use the same $3 \times 8 \times 5 = 120$ long–short or long-only sector neutral factors to estimate the likelihoods. For each type we form long–short or long-only standard factors and their sector-neutral versions. We bootstrap the time series of the factors 1,000 times and compute the difference in the Sharpe ratio of the standard factor and its sector-neutral version. This procedure provides us with 120,000 differences in Sharpe ratios for the long–short factor and 120,000 differences in Sharpe ratios for the long-only factor.

Figure A1 shows the distributions associated with this historical simulation. The larger standard deviations of the moments of the long–short factors are reflected in the wider distribution of the long–short outcome. This means that the long–short investor is more likely to gain or lose a substantial amount by hedging out sector bets. The total likelihood of the investor to lose, as reflected by the density mass of the red distribution that lies to the left of zero, is 20%; our back-of-the-envelope estimate for this probability was 29%.

The blue distribution shows the distribution of Sharpe ratio differences—as a result of removing sector bets—for the long-only portfolio. This distribution is narrow and peaked, consistent with the low volatility of the moments of long-only portfolios. The density mass on the negative side is 78%, which is precisely equal to its back-of-the-envelope predicted probability of 78%.

Table A1: Moments of data

The table shows the mean and standard deviations (SD) for the determinative moments of data for the inequality of (4). The mean and standard deviation (SD) are obtained using the data for $3 \times 8 \times 5 = 120$ long–short or long-only factor portfolios.

	Long-short			Long-only		
	ρ	SR_{within}	SR_{across}	ρ	SR_{within}	SR_{across}
Mean	0.465	0.154	0.032	0.794	0.158	0.149
SD	0.261	0.078	0.069	0.102	0.020	0.017

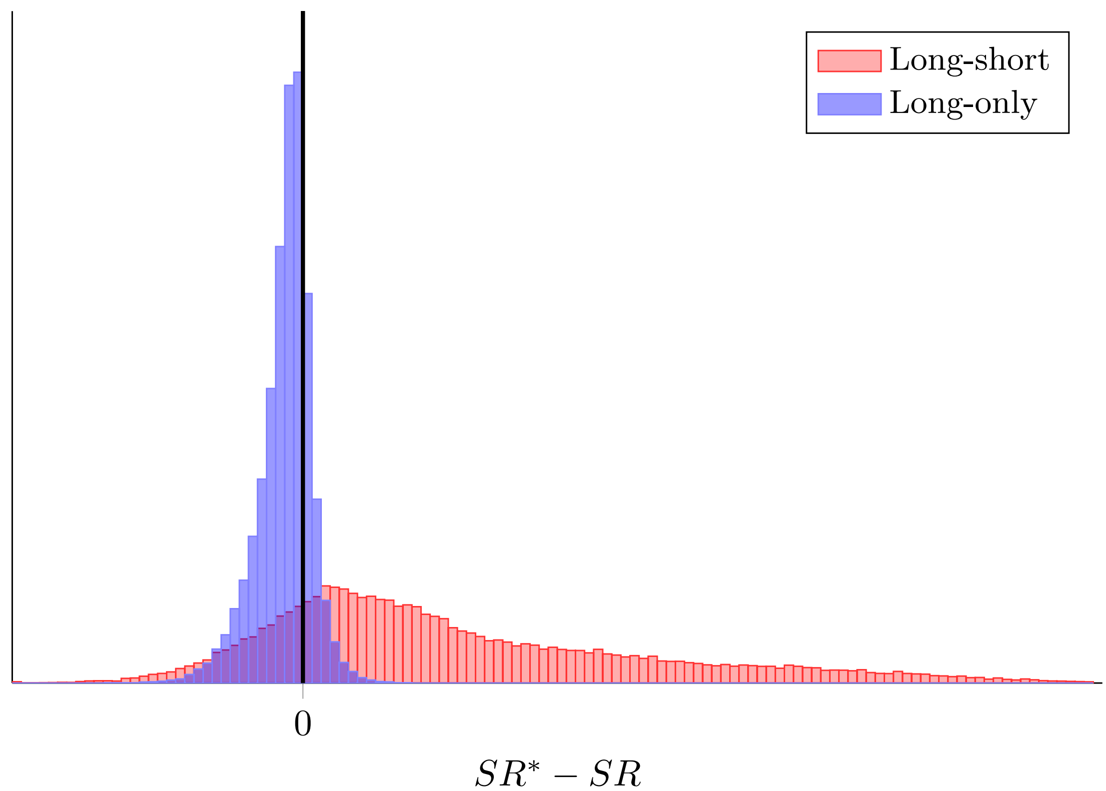

Figure A1: Historical simulation.
This figure shows the distribution for changes in Sharpe ratios of factors as a result of removing sector bets from historical data. The factors are size, value, profitability, investment, and momentum ($\times 5$). We construct equal-, rank-, and value-weighted factors ($\times 3$). We construct sector-neutral factors using eight definitions for sector classification based on the Fama and French industry classification ($\times 8$). This procedure generates $5 \times 3 = 15$ factors and $5 \times 3 \times 8 = 120$ sector-neutral factors. We form both long-only and long–short versions of the factors. The figure shows the distribution of differences between the Sharpe ratios of each sector-neutral factor and its original version. The distribution is obtained by bootstrapping the data by month.