Horizon Pricing

Avraham Kamara, Robert A. Korajczyk, Xiaoxia Lou, and Ronnie Sadka*

April 11, 2015

Forthcoming: *Journal of Financial and Quantitative Analysis*

*Kamara: Foster School of Business, University of Washington, Seattle, WA 98195-3226; email: *kamara@uw.edu.* Korajczyk: Kellogg School of Management, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208-2001; email: *r-korajczyk@kellogg.northwestern.edu.* Lou: Lerner College of Business and Economics, University of Delaware, Newark, DE 19716; email: *lous@udel.edu.* Sadka: Carroll School of Management, Boston College, Department of Finance, 140 Commonwealth Ave., Chestnut Hill, MA 02467; email: *sadka@bc.edu.* We would like to thank Yakov Amihud, Pierluigi Balduzzi, Stephen Brown, Mikhail Chernov, Kent Daniel, Ian Dew-Becker, Wayne Ferson, Carole Gresse, Avner Kalay, Maja Kos, Marc Lipson, Alan Marcus, Pedro Matos, Michael Schill, Avanidhar Subrahmanyam, an anonymous referee, and seminar participants at American Finance Association meetings, Babson College, Boston College, Center for Accounting Research and Education Conference, Chinese University of Hong Kong, Cirpée Applied Financial Time Series Workshop (HEC Montreal), DePaul University, Deutsche Bank Quant Conference, European Financial Management Association, Conference of Financial Economics and Accounting, George Washington University, Hedge Fund Research Conference (Paris), Inquire Europe Seminar, London Business School, London School of Economics, Q-Group Seminar, Rice University, SAC Capital Advisors, State Street Global Advisors, Tel Aviv University, University of Connecticut, University of Illinois, University of Lancaster, University of Manchester, University of Melbourne, and University of Virginia for comments.

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Abstract

The literature documents heterogeneity in the delay of stock-price reaction to systematic shocks, implying that asset risk depends on investment horizon. We study the pricing of risk factors across investment horizons. Value (liquidity) risk is priced over intermediate (short) horizons. Conditioning horizon-factor exposures on firm characteristics indicates that characteristics, with the exception of momentum, are not priced beyond their contribution to systematic risk. Long-horizon institutional investors overweight assets with high intermediate-horizon exposures to HML risk and high short-horizon exposures to liquidity risk. The results highlight the importance of investment horizon in determining risk premia.

I. Introduction

A number of asset-return and macroeconomic variables have been proposed in the asset-pricing literature as systematic priced risk factors—e.g., market risk (MKT, e.g., Treynor (1962), (1999)), Sharpe (1964), Lintner (1965)), size and value (SMB and HML, respectively, e.g., Fama and French (1993)), momentum (UMD, e.g., Carhart (1997)), and liquidity (LIQ, e.g., Pástor and Stambaugh (2003)). This paper studies the role of return horizon in the pricing of systematic risk. Specifically, we examine whether there exist factors whose risk measured over one horizon explains the cross-sectional differences in expected returns while their risk measured over alternative horizons does not.

Investment horizon has at least two potential connotations. One is the length of time over which the utility of consumption is defined. Horizon might be the lifetime of an individual investor or, possibly, be infinite for a charitable trust or individual investors with bequest motives for their heirs. Another notion is the length of time between consumption and portfolio rebalancing decisions. In single-period models, such as the capital asset pricing model (CAPM) these horizons coincide, although the theory is silent on the length of a period. Barberis (2000) and Campbell and Viceira (2002, Chapter 2) discuss the effect of lengthening the lifetime horizon on asset allocation in this setting. Alternatively, models might have very long lifetime horizons, but infinitesimal consumption-rebalancing horizons (e.g., Merton (1973)).

We postulate that different investors may have different rebalancing horizons. Empirically, systematic risk is dependent on the horizon of returns used to measure factor betas.

Therefore, investors' view of systematic risk of a given asset may be horizon dependent, leading to a clientele effect in which one horizon clientele underweights certain assets, causing another clientele to overweight those assets. In the extreme, there might be no overlap between the assets held by different horizon clienteles, hence different assets may be priced with different pricing kernels. This raises the question of whether some systematic risk factors are of more concern to one horizon clientele than another and, if so, whether the appropriate return horizon for measuring systematic risk differs across factors.

We explore the term structure of factor horizon premia by forming portfolios based on betas measured using returns over alternative horizons. We find that high-liquidity-beta portfolios have higher average returns than low-liquidity-beta portfolios when betas are estimated using horizons between one and six months (depending on the specification). MKT behaves like a 6-12 month risk factor, but the significance of its pricing is sensitive to experimental design. The HML factor is priced when betas are estimated using horizons of two to three years. The beta risks of the factors SMB and UMD do not appear to be priced.

A related question is whether firm characteristics, such as firm size and book-to-market equity, explain the cross section of returns because systematic risk is a function of these characteristics or because the characteristics explain variation in returns that is independent of the covariance structure of returns. For example, Daniel and Titman (1997) find that the data are more consistent with size and book-to-market being priced characteristics than risk factors. To investigate this, we include both characteristic values and historical factor betas in cross-sectional return regressions (Fama and MacBeth (1973)). Intermediate-term HML risk and short-term LIQ risk continue to be priced while controlling for the book-to-market, size, and momentum characteristics. Additionally, the characteristics have significant explanatory power for the cross section of returns.

However, the use of unconditional, rolling OLS estimates of betas in the analyses above may bias the results in favor of classifying size, book-to-market, and momentum as characteristics rather than risk factors. This can happen if these firm-specific variables help predict firms' true betas over and above the information included in the unconditional estimates. For example, a standard leverage effect suggests that changes in beta will be related to changes in size for levered firms. Therefore, firm size and return momentum might help us predict conditional betas, over and above the predictive power of unconditional betas. To insure that the cross-sectional predictive power of the characteristics is not due to their ability to predict betas, we apply a conditioning-variable approach (e.g., Ferson and Harvey (1997)). We estimate betas conditional on predetermined firm characteristics and use these conditional betas to estimate factor risk premia. In the conditional model, the characteristics have no explanatory power for returns beyond their ability to predict betas, with the exception of momentum.

The heterogeneity of investor horizon implies that some investors would underweight assets that are high-risk for their investment horizon, while others, with different investment horizons over which these assets appear relatively less risky, would overweight those assets in exchange for compensation for holding less-diversified positions. We find that long-horizon institutional investors overweight assets with high intermediate-horizon exposures to HML risk and high short-horizon exposures to liquidity risk. Therefore, these investors appear to be the bearers of priced systematic risk.

We discuss the robustness of our results to several possible concerns. First, (motivated by the results of (Chordia, Subrahmanyam, and Tong (2014)), the pricing of short-run liquidity risk and intermediate HML risk is not driven by an illiquid subset of stocks. We sort stocks into liquid and illiquid stocks based on their prior-year Amihud liquidity measure (Amihud (2002)) and estimate risk premia separately for each group. The evidence for the pricing of short-run liquidity risk and intermediate HML risk is generally stronger in the liquid stock sample.

Second, our results are not driven purely by an errors-in-variables (EIV) problem. Since longer-horizon betas are calculated with fewer effective degrees of freedom and, hence, less precision, the EIV problem would bias downward the risk premia estimated using longer-horizon risk measures. However, we find stronger evidence that HML is priced at intermediate horizons than at short horizons, which is inconsistent with the hypothesis that the EIV problem drives our results.

Third, our results are not purely driven by nonsynchronous trading. Nonsynchronous trading suggests that betas estimated using longer horizons are less biased and may yield more accurate risk premia estimates. While this explanation is consistent with the results for MKT and HML, it is not consistent with the results for the liquidity factor, which is priced only for relatively short horizons. Therefore, nonsynchronous price observations cannot explain the stronger pricing of the liquidity factor at short horizons.

Our results highlight the potential importance of investor horizon. If investors have heterogeneous investment horizons, for example, with long-run investors being less sensitive to shocks to shorter-horizon factors, then risks that appear systematic from a short-run perspective may not appear so in the long run. In this case, long-run investors can reap the risk premia associated with short-run factors without bearing, or with bearing less of, these risks in the long run. Hence, they are the natural bearers of those risks. For example, highly leveraged hedge funds that rely on short-term financing are likely to be concerned with short-horizon liquidity shocks since they may be forced to engage in fire sales precisely at times when these assets are the least liquid due to either the tightening of financing conditions or investor capital redemptions (e.g., Long Term Capital Management (Jorion (2000)), the quant crisis of 2007 (Khandani and Lo (2007), (2011)), and the financial crisis of 2008–2009). In contrast, other investors, such as pension funds, endowments, closed-end mutual funds, and long-run individual investors, have the ability to avoid trading in periods of temporary illiquidity. If there are horizon clienteles across investors, it may be necessary to measure systematic risk at different horizons for different factors.

II. Horizon, Risk, and Clienteles

A. Rational Inattention and Rebalancing Horizon

There is evidence that rebalancing horizon varies across investors and over time. While some investors choose to trade frequently and may be concerned with risk over short horizons, others may choose to trade infrequently due to costs of monitoring their portfolios and trading costs (Duffie and Sun (1990), Abel, Eberly, and Panageas (2007), (2013), Duffie (2010)) and may be concerned with risk over longer horizons. In particular, Abel, et al. (2013) derive a model in which investors face proportional and fixed costs of rebalancing their portfolio in addition to a utility cost of observing the state of their portfolio. In general, the horizon chosen to observe and rebalance a portfolio is state-dependent and, hence, an investor's horizon is stochastic. However, for fixed rebalancing costs that are sufficiently small, an optimally inattentive investor's strategy is purely time-dependent with a fixed horizon. Empirically, many investors seem to have long rebalancing horizons. Ameriks and Zeldes (2004) find that, for a sample of defined contribution retirement plan participants, 47% (21%) made no changes (one change) to their allocation of contributions over a ten-year period. Similar results are found for 401(k) plans by Agnew, Balduzzi, and Sundén (2003) and Mitchell, Mottola, Utkus, and Yamaguchi (2006). Chakrabarty, Moulton, and Trzcinka (2013) find a large amount of heterogeneity in the holding periods for equities held by a large sample of institutional funds.

At the opposite end of the spectrum from long-horizon investors are investors who trade frequently, such as institutional investors with levered portfolios subject to margin calls. The funds affected by the "Quant Meltdown" of 2007 (Khandani and Lo (2007), (2011)) provide examples of investors who trade at short horizons and care very much about shocks to asset prices that may be transitory.

B. Return Dynamics and Horizon-Dependent Systematic Risk

There is a long line of research suggesting that there is a delay in the reaction of prices of certain stocks to news about systematic factors (e.g., Lo and MacKinlay (1990), Brennan, Jegadeesh, and Swaminathan (1993), Badrinath, Kale, and Noe (1995), and Zhang (2006)). Several studies investigate the premise that market participants need more time to process the implications of shocks to complicated or opaque firms than they need for transparent firms. For example, Hou and Moskowitz (2005) report that delays in information processing account for part of several widely studied asset-pricing anomalies, while Hou (2007) shows that differences in speed of information processing are a leading cause of the lead–lag effect in intra-industry returns. Cohen and Lou (2012) document that monthly returns of focused, or easy-to-analyze, firms (i.e., firms that operate solely in one industry) incorporate industry-specific shocks faster than returns of complicated firms (i.e., conglomerates with multiple operating segments). As a result, monthly returns of easy-to-analyze firms predict the returns of more-complicated, within-industry, peers. Liu (2014) shows that stock prices of firms at the periphery of a network lead prices of central firms. Gilbert, Hrdlicka, Kalodimos, and Siegel (2014) find that CAPM betas of opaque firms are higher when using monthly returns instead of daily returns, whereas betas of transparent firms exhibit the reverse pattern. Duffie (2010) formalizes some of these ideas in a model wherein search costs create trade delays that result in delayed price reactions to shocks. Real options models (Hackbarth and Johnson (2014)) suggest that costly adjustment of capital causes the firm to wait longer between investment adjustments and leads to autocorrelation in its returns. Taken together with the idea that market participants face costly information processing, this can result in longer delays in the reactions of the share prices of these firms to news. In addition, costly adjustment of capital also implies that firms are less likely to react to a temporary productivity shock and may wait for a sequence of shocks before adjusting their capital.

Additionally, price delay may be caused by non-synchronous trading (e.g., Scholes and Williams (1977), Dimson (1979), and Cohen, Hawawini, Maier, Schwartz, and Whitcomb

(1983)). Delay in price reaction implies that systematic risk of a given asset will differ across investors' investment horizons. Even if risk factors are serially uncorrelated and without delays in price reactions, investment horizon may impact the appropriate measure of risk. For example, Levhari and Levy (1977) show that discreet compounding leads to estimates of systematic risk that are biased when estimated at horizons different from the horizon at which a single factor asset-pricing model holds.

The horizon effect on systematic risk is illustrated in Figure 1, in which we plot the betas of size decile portfolios relative to the value-weighted market portfolio, as a function of the return horizon used in estimating beta. The size decile and market portfolio returns are monthly returns from the Center of Research in Security Prices (CRSP) stock data base over the period January 1926 to March 2014. For a k-month horizon we compound the monthly decile and market portfolio returns over a k-month horizon and regress the k-month overlapping decile returns on the k-month overlapping market returns. Figure 1 shows that the estimated systematic risk can vary considerably with horizon, particularly for smaller market capitalization firms. For the smallest size decile the estimated market beta is 1.49 at a 1-month horizon and 1.97 at a 12-month horizon.

C. Horizon and Factor Risk Premia

Several empirical studies have shown that risk and risk premia estimates depend on horizon. Roll (1981) suggests that the difference in short-horizon and long-horizon beta estimates might explain the size effect of Banz (1981). Handa, Kothari, and Wasley (1989) find that the premium associated with market risk is insignificant when beta is estimated over monthly horizons but significant when beta is estimated over annual horizons and that the annual market beta drives out the size effect. Kothari, Shanken, and Sloan (1995) find a significant market premium using betas estimated over annual horizon, but they also find that the annual market betas do not subsume the size effect. Daniel and Marshall (1997) find that long consumption horizons do a better job in explaining the equity risk premium and risk-free rate puzzles in a model with habit formation. Jagannathan and Wang (2007) find that the Consumption Capital Asset Pricing Model (CCAPM) does a much better job explaining the cross section of asset returns using an annual horizon than using a quarterly horizon. In particular, an annual horizon ending in the fourth quarter has much higher explanatory power than annual periods ending in the other quarters. They suggest that investors tend to plan their consumption and investment choices in the last quarter of the year.

Brennan and Zhang (2013) and Beber, Driessen, and Tuijp (2012) estimate asset-pricing models that allow for various investment horizons and find that the cross-section of expected returns is better explained by risk measured at long horizons.

In Brennan and Zhang (2013) investors face a stochastic liquidation horizon. All investors are ex ante identical, so there are not horizon clienteles. There is no portfolio rebalancing before liquidation. Everyone holds the market portfolio, so an asset's beta relative to the market is the relevant measure of risk. However, the relevant beta for pricing assets is a weighted average of betas at different horizons, where the weights reflect the probability of liquidation at that horizon. Their specification provides an estimate of the probability-weighted liquidation horizon, which is 12.1 months. When the sample period is split in half the weighted horizon is 16.7 months in the first half of the sample (1926-1962) and 2.4 months in the second half of the sample (1963-2010), consistent with the notion that transactions costs have decreased and turnover has increased over time.

Beber, et al. (2012) derive an equilibrium pricing model in which investors have different, exogenously specified, investment horizons. Transactions costs are stochastic and differ, in expectation, across assets. The model leads to segmentation in which short horizon investors choose to avoid the most illiquid assets. Longer horizon investors must hold those assets. In addition to normal market risk, the expected return on an asset depends on liquidity premia, segmentation risk premia, and spillover risk premia. The segmentation premia are compensation for the imperfect risk sharing caused by the endogenous clienteles. Long-horizon (short-horizon) investors hold more (less) of the illiquid assets than they would in a no-transaction cost world. The spillover premia depend on the correlation between illiquid and liquid assets. For example, if there exist liquid assets that are perfectly correlated with the illiquid assets, then the long-horizon investors can hedge the segmentation risk in the liquid asset market. In this extreme case the segmentation premia and the spillover premia offset each other. In Beber, et al. (2012), dividends and transactions costs are independent and identically distributed (i.i.d.), implying that the horizon effect on risk, studied in Brennan and Zhang (2013) is not present here.

We have in mind a world in which there are two sources of potential clienteles. One source is the cross-sectional dispersion in expected transactions costs, as in Beber, et al. (2012). The other is difference in perceived systematic risk across investor horizons. Due to the return dynamics discussed in the previous section, asset i might have a higher long- versus short-horizon beta and asset j might lower long- versus short-horizon beta. This makes i (j) relatively more (less) appealing to short-horizon investors. This effect might also differ across risk factors.

To the extent that horizon affects the measurement of systematic risk, it is sensible that it might affect the measured risk premia of risk factors. In this paper, we examine whether there exist factors whose risk measured over one horizon explains the cross-sectional differences in expected returns while their risk measured over alternative horizons does not.

III. Data and Factors

Our sample consists of all NYSE/AMEX/NASDAQ-listed stocks whose price is above $1 at the beginning of each month. The stock price and return data are from the Center of Research in Security Prices (CRSP). We use Compustat industrial annual files to compute book value of equity. We follow Fama and French (2001, Appendix A.1) to compute book value of equity (BE). Our sample period is August 1962 to December 2013. Overall, our sample includes 17,166 unique firms, ranging from 1,281 to 5,742 firms per year, with an average of 3,741 per year.

We obtain monthly MKT, SMB, HML, and UMD factors data from Ken French's web site (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french). The monthly MKT factor, the excess return on the market, is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Morningstar (2014)). The monthly Fama–French SMB and HML factors are constructed using the six value-weighted portfolios formed on size and book-to-market (two-by-three sort). The UMD factor is the average return on the two high past-return portfolios minus the average return on the two low past-return portfolios. Details on the factor portfolio construction are available on Ken French's web site. The monthly Pástor and Stambaugh (2003) liquidity data, including the level of market liquidity and a nontraded liquidity factor, are from Ľuboš Pástor's web site.[1]

Factors of horizon k are constructed from the monthly factors. Note that each of the traded factors represents an excess return portfolio. Our k-period excess returns are constructed as the difference in the k-period returns of the long and short portfolios. For example, MKT return of horizon k is the k-period return of the market portfolio minus the k-period return of the risk-free asset ($f_{k,t}^{\text{MKT}} = \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^{m}) - \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^{f})$), where $r_{1,\tau}^{m}$ and $r_{1,\tau}^{f}$ are the monthly returns for the market portfolio and risk-free asset in month τ. Similarly, SMB of horizon k is the k-period return of the small capitalization portfolio minus the k-period return of the large capitalization portfolio. That is, $f_{k,t}^{\text{SMB}} = \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^{s}) - \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^{b})$, where $r_{1,\tau}^{s}$ and $r_{1,\tau}^{b}$ are the monthly returns for the small cap portfolio and large cap portfolio in month τ. We define the liquidity factor of horizon k in month t as the realized market liquidity level in month t, less its expected value at month $t - k$. To compute the expected liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2013, and the expected market liquidity in month t of horizon k is the k-month-ahead forecasted market liquidity at month $t - k$.

[1] http://faculty.chicagobooth.edu/lubos.pastor/research/.

IV. Factor Dynamics: The Term Structure of Factor Volatilities

We begin our empirical investigation by studying the volatilities of the factors (MKT, SMB, HML, UMD and LIQ) over different horizons. For factors that are portfolio excess returns, continuously compounded k-period excess returns are the sum of one-period excess returns over k periods. Under the null hypothesis that factor returns are uncorrelated across periods, the variance of k-period returns is k times the one-period variance. If factor returns are reinforcing (positively serially correlated), then the variance of k-period returns is greater than k times the one-period variances and, therefore, risk is larger for longer-horizon investors. Conversely, if factor returns are transitory (negatively serially correlated), then the variance of k-period returns is less than k times the one-period variances and risk is smaller for longer-horizon investors. We begin the study of factor dynamics by calculating variance ratios. A k-period variance ratio, VR(k), is defined as the ratio of variance of the factor over a k-period horizon and k times the variance at the one-period horizon:

$$\text{(1)} \qquad \mathrm{VR}(k) \;=\; \frac{\mathrm{var}(r^c_{k,t})}{k \cdot \mathrm{var}(r^c_{1,t})},$$

where $r^c_{k,t}$ is the k-month, continuously compounded excess return for period $t-k$ to t for traded factors (MKT, SMB, HML, and UMD) and is the unexpected component, conditional on observations up to time $t-k$, for the nontraded factor, LIQ. For example, $r^{c,\mathrm{MKT}}_{k,t} = \ln[\prod_{\kappa=1}^{k}(1+r^m_{1,t-k+\kappa})] - \ln[\prod_{\kappa=1}^{k}(1+r^f_{1,t-k+\kappa})]$, and $r^{c,\mathrm{SMB}}_{k,t} = \ln[\prod_{\kappa=1}^{k}(1+r^s_{1,t-k+\kappa})] - \ln[\prod_{\kappa=1}^{k}(1+r^b_{1,t-k+\kappa})]$. Autocorrelation in factor returns at the one-month horizon could induce large or small long-horizon effects depending on the sign of the autocorrelation. Therefore, persistent returns would generate variance ratios greater than or equal to one, while transitory returns would generate variance ratios below one (Campbell, Lo, and MacKinlay (1997, pp. 48–55)). The variance ratio is determined by the return autocorrelations, $\rho(\kappa)$, up to lag $k-1$:

$$\text{VR}(k) = 1 + 2\sum_{\kappa=1}^{k-1}\left(1 - \frac{\kappa}{k}\right)\rho(\kappa). \tag{2}$$

Table 1 reports several variance ratios, for a number of horizons ranging from one month to 60 months, of the different factors considered here and p-values from two-sided t-tests of the null hypothesis that $\text{VR}(k) = 1$. Figure 2 plots the variance ratios up to 60-month horizons. We postulate that liquidity (by construction) and momentum (Jegadeesh and Titman (1993) and Yao (2012)) might be expected to behave like short-horizon factors.

For the liquidity factor, $\text{VR}(2) = 0.50$ and $\text{VR}(36) = 0.03$. The momentum factor variance ratios are humped shaped with VR above 1.0 for horizons between two and 11 months. The ratio then drops to 0.82 at two years and continues to drop at longer horizons. However, none of these variance ratios are significantly different from unity. The market and HML factors have humped-shaped variance ratios with the ratios being at or above 1.0 at all horizons for MKT and HML. While the market portfolio's variance ratio is never significantly different from 1.0, the variance ratios for HML are significantly above 1.0 (at the 5% level) for all horizons between two months and 23 months. SMB has variance ratios that are consistently above 1.0 with a peak at 55 months. In contrast to HML, the variance ratios of SMB are not significantly different from 1.0 for any horizon.

V. Horizon Pricing

The analyses thus far seem to suggest that some factor risks may be of more concern to long-run investors while others to short-run investors. In this section, we first provide an anatomy of horizon betas and then study the pricing of different factors as a function of the investment horizon.

A. Delayed Price Reaction and an Anatomy of Horizon Beta

The heterogeneity across stocks in price reaction to news, discussed in Section II.A above, implies that the measured systematic factor risk of an asset will depend on the return horizon used to estimate risk. To illustrate the effects of price delay on systematic risk, we derive the relation between beta calculated at horizon k, β_k, and beta calculated at horizon 1, β_1. For simplicity, we focus on a single-factor model, and for ease of exposition, we use continuously compounded returns

$$r_{1,t} = a + \beta_1 f_{1,t} + \varepsilon_{1,t}. \tag{3}$$

Given the evidence for delayed reaction of prices of certain stocks to news about systematic factors discussed above, we allow $\varepsilon_{1,t}$ to be correlated with $f_{1,t-j}$. It follows that

$$\beta_k = \frac{\text{cov}(r_{k,t}, f_{k,t})}{\text{var}(f_{k,t})} = \frac{\text{cov}(\sum_{j=0}^{k-1} r_{1,t+j}, \sum_{j=0}^{k-1} f_{1,t+j})}{\text{var}(f_{k,t})}. \tag{4}$$

Given the expression for return, (3), we arrive at

$$\beta_k = \frac{\mathrm{cov}(\sum_{j=0}^{k-1} \beta_1 f_{1,t+j} + \varepsilon_{1,t+j}, \sum_{j=0}^{k-1} f_{1,t+j})}{\mathrm{var}(f_{k,t})} = \beta_1 + \frac{\mathrm{cov}(\sum_{j=0}^{k-1} \varepsilon_{1,t+j}, \sum_{j=0}^{k-1} f_{1,t+j})}{\mathrm{var}(f_{k,t})}. \tag{5}$$

Note that unless $\mathrm{cov}(\sum_{j=0}^{k-1} \varepsilon_{1,t+j}, \sum_{j=0}^{k-1} f_{1,t+j}) \neq 0$, $\beta_k = \beta_1$ independent of horizon and independent of the variance ratio of the factor. This suggests that the dynamic structure of the factor alone is insufficient for explaining systematic differences in betas across horizons. To explain differences in systematic risk across horizons, one needs to consider some form of delayed reaction of stock returns to the factor.

For simplicity, consider a one-period delayed reaction of the stock return to the factor, that is, $\mathrm{cov}(\varepsilon_{1,t}, f_{1,t-j}) \neq 0$ for $j = 1$, and zero for all other values of j. Then it follows that

$$\beta_k = \beta_1 + \frac{k-1}{k} \cdot \frac{1}{\mathrm{VR}(k)\mathrm{var}(f)} \cdot \mathrm{cov}(\varepsilon_{1,t}, f_{1,t-1}). \tag{6}$$

This expression has several implications. First, for a given firm, beta can vary with horizon as a function of delayed reaction, the factor variance ratio, and k. Second, even a delayed reaction of one period can induce difference of betas over periods longer than one period. Third, since firms differ in the extent of the delayed reaction of their stock price, the distribution of betas may change with horizon. That is, firms' beta ranking in the cross section can change with horizon. This can explain why sorting firms into different decile portfolios can produce different portfolios depending on the horizon by which the betas are calculated.

Additionally, the use of discrete rather than continuous compounding (as in Levhari and Levy (1977)) can lead to additional horizon effects in betas beyond those modeled above. However, our empirical analyses do not find these effects to be dominant.

B. Portfolio Returns

In this section, we form value-weighted portfolios based on preranking betas for each of the five factors at the end of each year and examine the monthly return spread between the highest beta decile and the lowest beta decile for the subsequent year. Betas are estimated for various horizons (k) ranging from one month to 61 months using overlapping k-month excess returns ($r^e_{k,t}$) and factors (e.g., $f^{\text{MKT}}_{k,t}$) in the five years prior to the portfolio-formation year. We estimate betas using a five-factor model—the Fama–French factors (MKT, SMB, and HML), plus the momentum factor, UMD, and the liquidity factor, LIQ. Our pricing tests are from January 1965 through December 2013 because our liquidity risk time series begins in August 1962 and we require at least 24 observations for beta estimation.

Table 2 reports the average (annualized) monthly excess returns for portfolios formed by independent sorts on each factor's beta. Additionally, the table gives alpha relative to the Fama–French four-factor model (MKT, SMB, HML, UMD) for liquidity-beta sorted portfolios. For example, the column labeled "β^{MKT}_k" is the annualized monthly excess return (in percent) of a portfolio that is long the highest preranking market beta decile and short the lowest preranking market beta decile. The corresponding t-statistics are in brackets. Similarly, the columns labeled "β^{LIQ}_k Return Spread" and "β^{LIQ}_k (FF4 alpha)" list the return spread and FF4 alpha for a portfolio long high Pástor–Stambaugh liquidity beta assets and short low liquidity beta assets.

We report the portfolio returns for horizons of 1, 3, 6, 12, 24, 36, 48, and 60 months. To increase power, we also use the portfolios corresponding to the adjacent horizons for horizons greater than one month. For example, to calculate the portfolio return spread of a one-year horizon, we use the portfolio returns of 11-, 12-, and 13-month horizons. That is, we average the returns of the three portfolios to create a time series of monthly excess returns, from which time-series average returns and corresponding t-statistics are computed.

The results in Table 2 show that liquidity beta has a significant premium at short horizons of three and six months, the market beta has a significant premium at horizons of six to 12 months, and HML beta has a significant premium at intermediate horizons of two to three years. The SMB and UMD betas do not exhibit any significant premia. In the last two columns we report the alphas (and t-statistics) of the liquidity-beta portfolios relative to the four-factor model (MKT, SMB, HML, and UMD). The data show that the liquidity-beta sorted portfolios earn significant abnormal returns at horizons of up to six months.

To summarize, the analysis in Table 2 highlights the different attributes of the factors at issue. Liquidity beta seems to capture a short-horizon risk. That is, liquidity risk measured using short-horizon data is priced, but liquidity risk measured using long horizons is not priced. In contrast, MKT and HML seem to behave like intermediate-horizon risk factors. That is, market and value/growth risk measured at monthly horizons are not priced, while market risk measured using six-month and annual horizons and value/growth risk measured using 24- and 36-month horizons are priced. The SMB and UMD betas do not seem to be priced at any horizon. Indeed, with the exception of one-month UMD beta, none of their

t-statistics is greater than one, in absolute value. Our results for MKT risk are similar to those in Handa, et al. (1989), Kothari, et al. (1995), Bandi, Garcia, Lioui, and Perron (2011), and Brennan and Zhang (2013), although the significant pricing of market beta is sensitive to changes in experimental design, as we document later. However, our results for the pricing of HML risk are novel. We are unaware of any study which documents HML risk by horizon.

To better understand which factors are priced in a given horizon, Figure 3 plots on one graph the average beta spread decile returns for each of the factors MKT, HML, and LIQ for each horizon. To smooth out the variations in average returns across close horizons, we average the premia across horizons every six months. For example, instead of plotting the average monthly return of MKT-beta spread separately using a one-month beta, a two-month beta, ..., and a six-month beta, we average these six average returns and use this average for horizons one through six. Figure 3 shows that the risk premium on MKT beta peaks at horizons of 6–12 months and falls substantially for longer periods. The risk premium on HML beta is higher for horizons of 12–36 months and falls substantially for periods longer than 48 months. Finally, the risk premium on LIQ beta is the highest for short horizons, of 1–6 months. It then falls substantially and tends to decline with horizon.

In light of the above-documented autocorrelation in returns, documented through the variance ratio structure of the risk factors, we would like to address the possibility that investors adjust their horizon-beta calculations for the potential predictability of the risk factors over horizons longer than one month. Specifically, we form pre-whitened horizon k factors ($f_{k,t}^{\text{MKT}}, f_{k,t}^{\text{SMB}}, f_{k,t}^{\text{HML}}, f_{k,t}^{\text{UMD}}, f_{k,t}^{\text{LIQ}}$) from the monthly factors as follows. The k-period excess returns are constructed as the difference in the k-period *unexpected* (instead of total) returns of the long and short portfolios. For example, $f_{k,t}^{\text{MKT}} = \prod_{\kappa=1}^{k}(1 + \Delta r_{1,t-k+\kappa}^{m}) - \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^{f})$; $f_{k,t}^{\text{SMB}} = \prod_{\kappa=1}^{k}(1 + \Delta r_{1,t-k+\kappa}^{s}) - \prod_{\kappa=1}^{k}(1 + \Delta r_{1,t-k+\kappa}^{b})$, where Δ denotes unexpected return. To calculate expected returns, we first determine the best univariate ARIMA model using the Bayesian information criterion (BIC) criterion (Schwartz (1978)) for each monthly portfolio return series (e.g., r_1^m, r_1^s) and the market liquidity level using the entire time series. The resulting models used are MA(1) for r^m, AR(1) for r^s, MA(3) for r^b, AR(3) for r^h, AR(1) for r^l, AR(1) for r^u, AR(3) for r^d, and AR(3) for the liquidity level. Expected returns are estimated each month $t-k$ using the data observed before the end of month $t-k$. The expected portfolio returns and market liquidity of horizon κ are the κ-month-ahead forecasted portfolio returns and market liquidity at month $t-k$. The unexpected portfolio returns (e.g.,$\Delta r_{1,t-k+\kappa}^{s}$) are the realized portfolio returns (e.g., $r_{1,t-k+\kappa}^{s}$) minus the expected portfolio returns.

Table 3 repeats the portfolio analyses of Table 2 using factors based on unexpected realizations (shocks). The results pertaining to liquidity risk are stronger than those reported in Table 2: in addition to the significant premia of liquidity beta at three and six months, the one-month beta is also significant (4.92% per year). The pricing results for the other factors (MKT, SMB, HML, and UMD) are also consistent with those in Table 2: the market beta has a significant premium at horizons of six to 12 months, HML beta has a significant premium at intermediate horizons of two to three years, and SMB and UMD betas do not exhibit any significant premia at any horizon. Overall, the results suggest that the horizon risk premia are robust to computing horizon factors using shocks instead of total return realizations. To maintain consistency with the risk factors as presented in the literature, the remaining analyses in this paper utilize the factors extracted from total returns.

C. Cross-Sectional Regressions

We study the robustness of our results by examining them using Fama and MacBeth (1973) cross-sectional regressions. To simplify and focus our analysis below we, henceforth, investigate the following nine combinations of factor exposures and horizons, which seem to be the most informative for our study: 1-, 6-, and 12-month MKT betas; 1- 12- and 24-month HML betas; and 1- 3- and 6-month LIQ betas.

In unreported analysis, the data suggest that the nine betas are quite different. Most of the average cross-correlations are small, suggesting that the length of the period over which we estimate the beta of each factor has a substantial impact on the ranking of stocks into decile portfolios.

Table 4 reports the results of the Fama–MacBeth regressions. We perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end; thus, the coefficients can be interpreted as value-weighted excess portfolio returns, similar in spirit to the value-weighted decile portfolio spreads studied above. To reduce the errors-in-variables problem, we replace a firm's beta with the average beta of the decile portfolio to which that firm is assigned based on the firm beta in month t. We report the time-series averages and t-statistics of cross-sectional coefficients, weighted by the inverse of the standard errors of the monthly coefficients (as in Litzenberger and Ramaswamy (1979)).

Column 1 of Table 4 reports the explanatory power for the size, book-to-market, and momentum characteristics, which are statistically significant. Columns 2–10 report the results using one of the nine betas above as the only risk variable. The average premia on the 24-month HML beta and 3-month LIQ beta are significantly positive at the level of 5% or less, while none of the MKT betas are significant at standard levels. Columns 11–13 report regressions, for each of the factors, using all of its three betas. For MKT, the 12-month horizon beta has the largest premium and t-statistic, although all three are insignificant at the 10% level. For HML, the 24-month horizon beta has the largest premium and is statistically significant at the 1% level, while the other two horizon betas are insignificant. For LIQ, the 3-month horizon beta has the largest premium and is statistically significant at the 10% level, while the other two horizon betas are insignificant. Column 14 reports regressions on the set of 12-month MKT beta, 24-month HML beta, and 3-month LIQ beta; both the 24-month HML beta and the 3-month LIQ earn significant (at the 5% level) positive premia.

In sum, liquidity risk seems to be priced at short horizons and HML risk is priced at intermediate horizons. The significant pricing of MKT risk at intermediate horizons, observed in Table 2, is not robust to inclusion of the size, book-to-market, and momentum characteristics. In what follows, in order to focus the study of horizons of different risk factors, we use the relevant horizon for each factor in light of the results of Table 4 and Figure 3.

VI. Beta or Characteristic?

The pricing of HML using intermediate-horizon betas and the lack of pricing of SMB using betas for any horizon between one and 60 months invites another look at the discussion about the pricing of characteristics versus betas (see Fama and French (1993), Daniel and Titman (1997), Davis, Fama, and French (2000)). Following the evidence in Fama and French (1992) showing that the characteristics size and book-to-market are priced in the cross section of stocks, Fama and French (1993) introduce the SMB and HML factors and argue that their respective betas price the cross section of size and book-to-market sorted portfolios. Daniel and Titman (1997) argue that once controlling for firm characteristics, the pricing of the Fama and French factors is unclear. Davis, et al. (2000), using a longer sample period, find evidence supporting the interpretation of HML as a risk factor. The evidence for SMB as a risk factor is less clear. We wish to study whether variables that behave like firm characteristics at one horizon behave like risk factors at another horizon.

Other than liquidity, the factors used in this paper (MKT, SMB, HML, and UMD) are formed as traded portfolio return spreads of high minus low beta deciles relative to the factors. It is therefore natural to study whether the pricing of the factor betas remains when controlling for the pricing of their respective firm characteristics for SMB, HML, and UMD.

Some initial insights can be drawn from Table 4. Column 1 of that table reports the results of cross-sectional regressions of returns on size, value (book-to-market equity ratio), and momentum (cumulative lagged return in months t-12 to t-2) as characteristics. Consistent with prior literature, these characteristics significantly predict the cross section of stock returns, where size has a negative coefficient and value and momentum have positive coefficients. Moving across Columns 2–14, these three characteristics seem relatively unaffected by the inclusion of factor betas in the regressions. Therefore, size, book-to-market, and momentum continue to behave as priced firm characteristics, with the exception that the book-to-market characteristic turns insignificant in the presence of intermediate-horizon HML betas.

However, the use of unconditional, rolling OLS estimates of betas in the analyses above may bias the results in favor of classifying size, book-to-market, and momentum as priced characteristics rather than risk factors. This can happen if these firm-specific variables help predict firms' true betas over and above the information included in the unconditional estimates (Ferson and Harvey (1997)). For example, a standard leverage effect suggests that changes in beta will be related to changes in stock price. Therefore, firm size and return momentum might help us predict conditional betas, over and above the predictive power of unconditional betas. Table 5 reports the results of Fama–MacBeth regressions using conditional betas with characteristics such as size and value as conditioning variables. All betas in the regression of month t are estimated using a firm's entire time series with size, book-to-market ratio, past returns, and historical beta as the conditioning variables. Specifically, for a given horizon k, we first estimate the following panel regression:

$$(7) \qquad r_{k,t}^{i,e} = a + b' Z_{t-k}^{i,\text{MKT}} f_{k,t}^{\text{MKT}} + s' Z_{t-k}^{i,\text{SMB}} f_{k,t}^{\text{SMB}} + h' Z_{t-k}^{i,\text{HML}} f_{k,t}^{\text{HML}} + m' Z_{t-k}^{i,\text{UMD}} f_{k,t}^{\text{UMD}} + l' Z_{t-k}^{i,\text{LIQ}} \text{LIQ}_{k,t} + \varepsilon_{k,t}^{i},$$

where $r_{k,t}^{i,e}$ is the cumulative excess return of stock i in the k–month interval $[t-k, t]$, $Z_{t-k}^{i,f} = (1, \text{Size}_{t-k}^{i}, \text{B/M}_{t-k}^{i}, r_{11,t-k-1}^{i}, \beta_{k,t-k}^{i,f})$ and $f = \{\text{MKT,SMB,HML,UMD,LIQ}\}$. The subscript

$t-k$ denotes that the variables are measured at time $t-k$. For example, Size_{t-k} is the logarithm of market cap measured at the beginning of month $t-k$; $r_{11,t-k-1}$ is the 11-month cumulative return between month $t-k-12$ and month $t-k-1$. B/M_{t-k} is the book-to-market ratio of month $t-k$. We use the book value of the fiscal year ending in year y-1, and market value in December of year y-1 for the 12 months from July of year y to June of year y+1. The variable $\beta^{i,f}_{k,t-k}$ is the most recent historical k-month OLS factor beta of the firm estimated with a Fama–French four-factor model, plus the liquidity factor, over the five years before month $t-k$. Vectors b, s, h, m, and l, are each 5×1 parameter vectors determining the relation between $Z^{i,f}_{t-k}$ and the factors.

All the conditioning variables in equation (7) are standardized to a mean of zero and a standard deviation of one in the cross section. We then use the coefficients estimated from the panel regression and the current realization of the Zs to generate conditional betas. In each month t, we then perform monthly weighted least square cross sectional regressions of returns on these conditional betas, where the weight is firm market capitalization at the previous month-end.

Table 5 contains the results. Reported are the time-series averages and t-statistics (in brackets) of cross-sectional regression coefficients, weighted by the inverse of the standard errors of monthly coefficients. As in Table 4, the market risk premium is insignificant, while HML-beta risk is significantly priced when betas are measured at the 24-month horizon, and liquidity risk is priced when betas are measured at the 3-month horizon.

In contrast to the unconditional results, the size and book-to-market characteristics are no longer significantly priced in Table 5 (with the exception of size in Column 2). In fact, the coefficient on the book-to-market characteristic is negative for every specification (in contrast to the typical positive relation), and even significant in Column 4. The results suggest that once firm-level size and book-to-market characteristics are incorporated into the factor betas as conditioning variables, they do not have a significant additional effect on expected returns. These results are consistent with the hypothesis that some of the predictive power of the size and book-to-market characteristics (evident in Table 4) is due to their ability to explain beta beyond the explanatory power of lagged OLS estimates of factor betas. The momentum characteristic effect however remains a statistically significant determinant of equity returns. Hence, while we cannot reject the hypothesis that conditional UMD-betas are not priced, we find consistent evidence that the momentum characteristic is priced.

VII. Investor Horizon

Above, we establish that different risk factors are priced over different horizons. In this section, we study the question: Who bears these risks and earns their premia—short-term or long-term investors? To answer this, we study the relation between the horizon of investors holding an asset, as measured by the frequency of turnover in their portfolio, the risk they undertake, as measured by horizon betas, and the pricing of these betas. The heterogeneity of investor horizon implies that some investors would underweight assets that are high-risk for their investment horizon, while others, with different investment horizons over which these assets appear relatively less risky, would overweight those assets in exchange for compensation for holding less-diversified positions. For example, in the model of Beber, et al. (2012) short-horizon investors wish to avoid liquidity risk and underweight assets with high liquidity risk. The long-run investors, therefore, must overweight the assets with high short-term risk and are paid a premium for holding an underdiversified portfolio.

We classify institutional investors as short-run, intermediate-run, or long-run by looking at the frequency with which they change their holding of stocks, as reported on their quarterly 13-F filings with the Securities and Exchange Commission (SEC). All institutions with more than $100 million under management are required to file their long portfolio holdings with the SEC. We obtain the data from the Thomson-Reuters Institutional Holdings (13F) Database. These holdings data are available starting January 1980. For each institutional investor, we calculate the portfolio's propensity to turnover shares through the Churn Ratio (Churn) as in Gaspar, Massa, and Matos (2005). Let q denote the quarter, i be an index of assets, and j be an index of institutional investors. $\text{Shares}_{i,j,q}$ is the number of shares of asset i owned by institution j at the end of quarter q, and $\text{P}_{i,q}$ is the price of stock i at the end of quarter q. The churn ratio for investor j and quarter q is

$$(8) \qquad \text{CHURN}_{j,q} = \frac{\sum_{i=1}^{n} |\text{Shares}_{i,j,q}\text{P}_{i,q} - \text{Shares}_{i,j,q-1}\text{P}_{i,q-1} - \text{Shares}_{i,j,q-1}\Delta\text{P}_{i,q}|}{\sum_{i=1}^{n} (\text{Shares}_{i,j,q}\text{P}_{i,q} + \text{Shares}_{i,j,q-1}\text{P}_{i,q-1})/2}.$$

The investor j turnover ratio for quarter q, $\text{CHURN}_{j,q,4}$, is the average of the quarterly churn ratio in the four quarters from quarter $q-3$ to quarter q. Figure 4 shows the evolution of various percentiles of the cross-sectional distribution of $\text{CHURN}_{j,q,4}$. There is a general upward trend in the churn ratio, which is more pronounced in the 75^{th} and 90^{th} percentiles.

We define investor j's horizon in quarter q, $\text{HORIZON}_{j,q}$, as the inverse of $\text{CHURN}_{j,q,4}$, divided by four (to annualize the statistic). Investors are classified as a long-run investor if their horizon is greater than, or equal to, three years and are classified as a short-run investor if their horizon is less than, or equal to, half a year. From Figure 4 one can see that the 10^{th} percentile of the churn ratio has a value of 0.05 per quarter, which implies a five-year investment horizon ($\frac{1}{0.05} = 20$ quarters).

For each stock-year (i,q) in our sample, we calculate the fraction of its shares held by long-run institutional investors measured at the end of quarter. This is the ratio of the shares of stock i held by long-run institutional investors and shares outstanding, $\omega^L_{i,q}$. Similarly, we calculate the fraction of firm i's stock held by medium- and short-run institutional investors, $\omega^M_{i,q}$ and $\omega^S_{i,q}$, respectively.[2]

To study the risk exposures of different institutional investors, we run Fama–MacBeth regressions of the ownership ratios on the horizon betas and firm characteristics. We use value-weighted regressions, run quarterly; standard errors are calculated using Newey–West standard errors with two lags. Specifically, for each quarter q between 1981 and 2013, we regress the ownership ratios measured at the beginning of the quarter on horizon betas that are estimated in the prior five years and three firm characteristics measured at the beginning of the quarter.

The results are reported in Table 6. We find that long-horizon institutional investors

[2]The 13-F data have a number of limitations. Since investors' positions are observed at a quarterly frequency, we are unable to identify gradations of horizons below one quarter. That is, we are unable to tell the difference between an investor who turns over the portfolio on a quarterly basis and a high-frequency trader who is turning over positions in microseconds. Additionally, the 13-F data do not include the short side of the investors portfolio (when they have short positions). If investors' long and short positions have different churn ratios, looking at the long positions only will give a biased estimate of churn and horizon. Finally, we do not have data on the horizon of the investors holding the stock who do not file Form 13-F, most notably individual investors. Gompers and Metrick (2001) show that the fraction of the equity market held by 13-F filers increased steadily from 28.4% in 1980 to 51.6% in 1996.

overweight assets with high intermediate-horizon exposures to HML risk and high short-horizon exposures to liquidity risk. In contrast, short-horizon institutional investors tend to underweight exposures to short-horizon liquidity risk. To insure that the results are driven by investor horizon rather than the pure effect of the level of institutional ownership, we also add an analysis where the dependent variable is the level of institutional ownership. Total institutional ownership is neither significantly related to HML betas nor to liquidity horizon betas. The results suggest that long-horizon investors are the natural holders of short-run and intermediate-run systematic risk.

VIII. Additional Tests

In an internet appendix to this paper we provide additions tests of the robustness of our results. We show that the results are not driven by illiquid assets. We rerun the results using longer holding periods for asset returns and the pricing results are unchanged. Finally, we estimate our cross-sectional regression with changes in betas across horizons and the results are consistent with those reported above.

IX. Conclusion

Delayed reaction of prices of stocks to news about systematic factors, found in a number of papers, implies that measured systematic risk will depend on the horizon over which returns are measured. Additionally, systematic factors that are portfolio excess returns tend to exhibit volatility at longer horizons that is greater than a proportionate scaling up of short-horizon volatility. Other priced factors, such as liquidity, are not persistent. This suggests that factor risk measured at short horizons might be more relevant for transitory factors since that may match the relevant horizon for short-horizon investors. Conversely, risk measured at longer horizons is more relevant for persistent factors.

We study a set of factors representing risks associated with shocks to the market, small- versus large-capitalization firms, value versus growth stocks, momentum stocks, and liquidity. Short-horizon (monthly) measures of risk seem to be important for the pricing of liquidity risk, consistent with its more transitory nature. Intermediate-horizon measures of risk are important for the pricing of market and value/growth risk. The value factor behaves like a characteristic when risk is measured at a monthly horizon and has both risk-factor and characteristic-like behavior at longer horizons (two years) when unconditional OLS betas are used. When we estimate a model that conditions betas on characteristics, HML is priced as a risk factor, while size and book-to-market have no additional explanatory power for the cross section of returns. Momentum remains significant as a priced characteristic.

The results highlight the importance of considering investment horizon in determining whether a cross-sectional return spread is alpha or a premium for systematic risk. Some factors that are risky from the perspective of short-run investors may not be so from the perspective of long-run of investors, and vice versa. In particular, liquidity risk may be of particular concern for short-horizon investors while presenting less long-horizon risk to others; whereas HML risk may be of particular concern for intermediate-horizon investors. Indeed, when we measure investor horizon using institutional ownership data, we find that long-run investors overweight assets with high short-horizon liquidity risk and high intermediate-horizon HML risk. Therefore, these long-run investors appear to be the natural bearers of systematic risk.

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Table 1: Factor Variance Ratios

A k-period variance ratio is defined as the ratio of variance of the factor over a k-period horizon and the product of k and the variance at the one-period horizon. $\mathrm{VR}(k) = \mathrm{var}(r^c_{k,t})/[k \cdot \mathrm{var}(r^c_{1,t})]$, where $r^c_{k,t}$ is the continuously compounded excess return for period t over a k-period horizon for traded factors and unexpected liquidity of horizon k for nontraded factor LIQ. Each traded factor (MKT, SMB, HML, and UMD) represents an excess return portfolio. For example, MKT is the market return in excess of the risk-free rate: $r^{c,\mathrm{MKT}}_{k,t} = \ln[\prod^k_{\kappa=1}(1+r^m_{1,t-k+\kappa})] - \ln[\prod^k_{\kappa=1}(1+r^f_{1,t-k+\kappa})]$; SMB is the return of small firms in excess of big firms: $r^{c,\mathrm{SMB}}_{k,t} = \ln[\prod^k_{\kappa=1}(1+r^s_{1,t-k+\kappa})] - \ln[\prod^k_{\kappa=1}(1+r^b_{1,t-k+\kappa})]$. The nontraded liquidity factor LIQ of horizon k in month t is the realized market liquidity level in month t, less its expected value at month $t-k$. To compute the expected value of liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2013, and the expected market liquidity level in month t of horizon k is the k-month-ahead forecasted market liquidity at month $t-k$. The sample period is 1963 through 2013.

	Panel A. Variance Ratio					Panel B. p-value. H_0: Variance Ratio = 1				
Months (k)	MKT	SMB	HML	UMD	LIQ	MKT	SMB	HML	UMD	LIQ
1	1	1	1	1	1					
2	1.10	1.07	1.17	1.05	0.50	0.08	0.25	0.00	0.34	0.00
6	1.21	1.08	1.42	1.01	0.18	0.10	0.53	0.00	0.91	0.00
12	1.27	1.19	1.54	0.98	0.09	0.15	0.32	0.00	0.93	0.00
24	1.29	1.38	1.51	0.82	0.05	0.29	0.16	0.06	0.51	0.00
36	1.19	1.55	1.27	0.73	0.03	0.57	0.10	0.42	0.43	0.00
48	1.12	1.69	1.15	0.75	0.02	0.75	0.08	0.70	0.52	0.01
60	1.18	1.71	1.12	0.71	0.02	0.68	0.11	0.78	0.51	0.03

Table 2: Pricing of Fama–French Factors and Liquidity Factor

At the beginning of each month in year y (y is from 1965 to 2013), stocks are sorted into 10 portfolios based on each of the five k-month betas (β_k^{MKT}, β_k^{SMB}, β_k^{HML}, β_k^{UMD}, β_k^{LIQ}), where k is from one to 61. The k-month betas are estimated using overlapping k-month (for $k > 1$) excess returns $r_{k,t}^e$ and overlapping k-month factors (e.g., $f_{k,t}^{\text{MKT}}$), where t denotes each month in the five years prior to the portfolios formation year y. The factors used in estimating betas include Fama–French three factors, UMD, and the Pástor–Stambaugh Liquidity Factor. Overlapping k-month cumulative excess return in month t, $r_{k,t}^e$, is defined as $\prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}) - \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^f)$. Factors of horizon k ($f_{k,t}^{\text{MKT}}, f_{k,t}^{\text{SMB}}, f_{k,t}^{\text{HML}}, f_{k,t}^{\text{UMD}}, f_{k,t}^{\text{LIQ}}$) are constructed from the monthly factors. Our k-period excess returns are constructed as the difference in the k-period returns of the long and short portfolios (for example, $f_{k,t}^{\text{MKT}} = \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^m) - \prod_{\kappa=1}^{k}(1 + r_{1,t-k+\kappa}^f)$). We define the liquidity factor of horizon k in month t as the realized market liquidity level in month t, less its expected value at month $t - k$. To compute the expected liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2013, and the expected market liquidity in month t of horizon k is the k-month-ahead forecasted market liquidity at month $t - k$. The table reports the average (annualized) monthly excess returns for independent sorts on each factor's beta, plus the alpha relative to the Fama–French four-factor model (MKT, SMB, HML, UMD) for liquidity-beta sorted portfolios. For example, the column labeled "β_k^{MKT}" is the monthly value-weighted excess return (in percent) of a portfolio that is long 10% of the assets with the highest preranking market beta and short 10% of the assets with the lowest preranking market beta. For brevity, we report the portfolio returns for horizons (k) of 1, 6, 12, 24, 36, 48, and 60 months. To increase power, to calculate the portfolio return spread of a one-year horizon, we use the portfolio returns of 11-, 12-, and 13-month horizons. That is, we average the returns of the three portfolios per month to create a time series of monthly excess returns, from which average returns and corresponding t-statistics (in brackets) are computed. Our sample includes all stocks that have k-month cumulative excess returns ($r_{k,t}^e$) for at least 24 of the 60 months ending in December of year y-1 and have a price of at least one dollar at the beginning of the portfolio formation month. The portfolio formation period is 1965 to 2013.

Horizon	β_k^{MKT}		β_k^{SMB}		β_k^{HML}		β_k^{UMD}		β_k^{LIQ}			
(k)	Return Spread		Return Spread		Return Spread		Return Spread		Return Spread		FF4 Alpha Spread	
1	0.41	[0.16]	-1.69	[-0.49]	2.00	[0.67]	-2.71	[-1.10]	2.89	[1.46]	4.82	[2.36]
[2,3,4]	2.69	[1.20]	-2.79	[-0.86]	1.92	[0.76]	-2.70	[-1.27]	3.84	[2.10]	4.44	[2.35]
[5,6,7]	5.68	[2.75]	-1.49	[-0.52]	1.32	[0.54]	0.19	[0.09]	4.04	[2.24]	4.34	[2.35]
[11,12,13]	4.08	[2.13]	-1.40	[-0.54]	3.56	[1.64]	-0.25	[-0.13]	0.25	[0.16]	0.84	[0.51]
[23,24,25]	1.03	[0.55]	1.84	[0.84]	4.78	[2.27]	0.52	[0.29]	-0.74	[-0.50]	-0.68	[-0.43]
[35,36,37]	1.64	[0.86]	-0.19	[-0.09]	4.72	[2.39]	-1.4	[-0.74]	1.04	[0.66]	0.88	[0.54]
[47,48,49]	1.47	[0.79]	1.28	[0.63]	1.54	[0.90]	-3.4	[-1.84]	-0.4	[-0.27]	-0.25	[-0.17]
[59,60,61]	0.13	[0.07]	2.22	[1.13]	1.56	[0.85]	-2.22	[-1.29]	0.34	[0.22]	-0.03	[-0.02]

Table 3: Constructing k-horizon Factors Using Shocks

This table repeats the analysis in Table 2 using an alternative factor construction procedure. Factors of horizon k ($f_{k,t}^{\text{MKT}}, f_{k,t}^{\text{SMB}}, f_{k,t}^{\text{HML}}, f_{k,t}^{\text{UMD}}, f_{k,t}^{\text{LIQ}}$) are constructed from the monthly factors. The k-period excess returns are constructed as the difference in the k-period *unexpected* returns of the long and short portfolios. For example, $f_{k,t}^{\text{MKT}} = \prod_{\kappa=1}^{k}(1+\Delta r_{1,t-k+\kappa}^{m}) - \prod_{\kappa=1}^{k}(1+r_{1,t-k+\kappa}^{f})$; $f_{k,t}^{\text{SMB}} = \prod_{\kappa=1}^{k}(1+\Delta r_{1,t-k+\kappa}^{s}) - \prod_{\kappa=1}^{k}(1+\Delta r_{1,t-k+\kappa}^{b})$, where Δ denotes unexpected return. To calculate expected returns, we first find the best ARIMA model using the BIC criterion for each monthly portfolio return series (e.g., r_1^m, r_1^s) and the market liquidity level except for the risk-free rate using the entire time series. The models are MA(1) for r^m, AR(1) for r^s, MA(3) for r^b, AR(3) for r^h, AR(1) for r^l, AR(1) for r^u, AR(3) for r^d, and AR(3) for the liquidity level. We then estimate these models at each month t-k using the data observed before the end of month t-k. The expected portfolio returns and market liquidity of horizon κ (i.e., in month $t-k+\kappa$) are the κ-month-ahead forecasted portfolio returns and market liquidity at month $t-k$. The unexpected portfolio returns (e.g.,$\Delta r_{1,t-k+\kappa}^{s}$) are the realized portfolio returns (e.g., $r_{1,t-k+\kappa}^{s}$) minus the expected portfolio returns.

Horizon	β_k^{MKT}		β_k^{SMB}		β_k^{HML}		β_k^{UMD}		β_k^{LIQ}			
(k)	Return	Spread	Return	Spread	Return	Spread	Return	Spread	Return	Spread	FF4 Alpha	Spread
1	1.16	[0.44]	-2.47	[-0.74]	1.01	[0.34]	-3.28	[-1.33]	4.92	[2.57]	4.14	[2.12]
[2,3,4]	3.11	[1.33]	-4.36	[-1.36]	2.60	[0.99]	-1.80	[-0.80]	3.77	[2.12]	3.93	[2.16]
[5,6,7]	5.55	[2.62]	-2.03	[-0.71]	2.16	[0.88]	-0.01	[0.00]	3.49	[1.98]	3.80	[2.12]
[11,12,13]	3.91	[1.96]	-1.25	[-0.49]	5.31	[2.44]	0.47	[0.24]	-1.10	[-0.63]	-1.08	[-0.61]
[23,24,25]	0.88	[0.46]	1.20	[0.54]	4.57	[2.13]	-0.02	[-0.01]	-1.57	[-1.03]	-1.57	[-0.99]
[35,36,37]	1.86	[0.96]	0.12	[0.06]	4.65	[2.30]	-0.95	[-0.50]	1.05	[0.68]	0.61	[0.38]
[47,48,49]	1.42	[0.74]	1.21	[0.61]	1.45	[0.83]	-3.04	[-1.62]	-0.73	[-0.47]	-0.74	[-0.47]
[59,60,61]	-0.32	[-0.17]	3.19	[1.60]	1.37	[0.73]	-2.72	[-1.49]	0.07	[0.05]	-0.33	[-0.22]

Table 4: Fama–MacBeth Regression Results

The table reports the results of Fama-MacBeth regressions. In each month t in year y, we perform value-weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. For all betas in the regression of month t, the average beta of the firms in the decile portfolio to which a firm is assigned based on that beta is used for the firm beta. The k-month betas are estimated using overlapping k-month (for $k > 1$) excess returns $r^e_{k,\tau}$ and overlapping k-month factors (e.g., $f^{\mathrm{MKT}}_{k,\tau}$), where τ denotes each month in the five years prior to year y. For example, β^{MKT}_1 is the market beta estimated using monthly returns in the years [y-5,y-1] for each month t in year y; β^{MKT}_6 is the market beta estimated using overlapping six-month cumulative returns. Variable Size is the natural logarithm of market cap measured at the end of month t-1. Variable B/M is the book-to-market ratio of month t. We use the book value of the fiscal year ending in year y-1 and market value in December of year y-1 for the 12 months from July of year y to June of year y+1. $r_{11,-2}$ is the 11-month cumulative return in months [t-12,t-2]. All independent variables are standardized to a mean of zero and a standard deviation of one in each month. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional regression coefficients, weighted by the inverse of the standard errors of monthly coefficients. Penny stocks are excluded and the sample period is 1965 through 2013.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
β_1^{MKT}		-0.03									-0.09			
		[-0.60]									[-1.56]			
β_6^{MKT}			0.02								0.003			
			[0.38]								[0.14]			
$\beta_{12}^{\mathrm{MKT}}$				0.04							0.06			0.03
				[0.90]							[1.10]			[0.61]
β_1^{HML}					0.06							0.02		
					[1.01]							[0.38]		
$\beta_{12}^{\mathrm{HML}}$						0.08						-0.05		
						[1.63]						[-0.91]		
$\beta_{24}^{\mathrm{HML}}$							0.13					0.13		0.13
							[2.89]					[2.92]		[2.65]
β_1^{LIQ}								0.05					0.04	
								[1.33]					[0.85]	
β_3^{LIQ}									0.09				0.09	0.09
									[2.24]				[1.84]	[2.09]
β_6^{LIQ}										0.04			-0.04	
										[0.91]			[-0.98]	
Size	-0.11	-0.11	-0.10	-0.11	-0.10	-0.10	-0.12	-0.11	-0.11	-0.10	-0.11	-0.11	-0.12	-0.10
	[-2.26]	[-2.23]	[-1.99]	[-2.13]	[-2.06]	[-2.26]	[-2.30]	[-2.11]	[-2.07]	[-1.98]	[-2.30]	[-2.35]	[-1.92]	[-2.21]
B/M	0.13	0.13	0.13	0.13	0.10	0.11	0.08	0.12	0.12	0.12	0.13	0.07	0.11	0.09
	[2.22]	[2.22]	[2.21]	[2.15]	[1.94]	[1.82]	[1.43]	[1.98]	[2.00]	[2.04]	[2.23]	[1.41]	[1.90]	[1.54]
$r_{11,-2}$	0.36	0.36	0.36	0.34	0.31	0.31	0.30	0.33	0.32	0.31	0.36	0.27	0.29	0.31
	[4.28]	[4.29]	[4.05]	[3.82]	[3.89]	[3.60]	[3.40]	[3.79]	[3.64]	[3.46]	[4.05]	[3.24]	[3.31]	[3.56]
Intercept	1.12	1.12	1.10	1.11	1.10	1.11	1.14	1.11	1.11	1.09	1.12	1.14	1.08	1.13
	[5.26]	[5.24]	[4.93]	[4.97]	[4.92]	[4.93]	[5.01]	[5.03]	[5.02]	[4.86]	[5.16]	[4.96]	[4.85]	4.983
Adjusted R^2	0.09	0.09	0.08	0.08	0.09	0.08	0.08	0.08	0.08	0.08	0.10	0.10	0.09	0.10

Table 5: Fama–MacBeth Regressions — Conditional Betas

The table reports the results of value-weighted Fama–MacBeth regressions using horizon betas conditional on firm characteristics as described in Section VI. All independent variables are standardized to a mean of zero and a standard deviation of one in each month. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional regression coefficients, weighted by the inverse of the standard errors of monthly coefficients. Penny stocks are excluded and the sample period is from 1965 to 2013.

	(1)	(2)	(3)	(4)
β_1^{MKT}	-0.12			
	[-1.70]			
β_6^{MKT}	0.03			
	[0.42]			
β_{12}^{MKT}	0.38			0.29
	[0.80]			[0.65]
β_1^{HML}		0.04		
		[0.44]		
β_{12}^{HML}		-0.05		
		[-0.49]		
β_{24}^{HML}		0.30		0.32
		[2.70]		[2.94]
β_1^{LIQ}			0.05	
			[0.52]	
β_3^{LIQ}			0.67	0.64
			[1.77]	[2.08]
β_6^{LIQ}			-0.06	
			[-0.98]]	
Size	-0.00	-0.11	-0.05	-0.02
	[0.00]	[-2.24]	[-0.77]	[-0.23]
B/M	-0.19	-0.18	-0.31	-0.89
	[-0.49]	[-1.75]	[-1.34]	[-2.26]
$r_{11,-2}$	0.48	0.33	0.76	0.94
	[2.26]	[3.59]	[2.36]	[3.51]
Intercept	1.13	1.14	1.09	1.15
	[5.19]	[4.95]	[4.86]	[5.03]
Adjusted R^2	0.10	0.10	0.09	0.10

Table 6: Holdings of Investors with Different Horizons

The table reports the quarterly value-weighted Fama-MacBeth regression results of ownership from institutions with different horizons on stock k-period betas and firm characteristics in each quarter q from 1981 to 2013. Using the churn ratio to measure investment horizon, we classify each institution as either short (horizon up to six months), intermediate, or long horizon (above three years). A stock's ownership by long-horizon (short-horizon) investors is the percentage of its shares owned by the long-horizon (short-horizon) investors. Total IO is the percentage of shares owned by all institutions. Horizon betas are estimated in the five years before quarter q; ownership ratios and firm characteristics are measured at the beginning of the quarter. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional regression coefficients, and T-statistics are calculated using Newey-West standard errors with two lags.

Dependent Variable	Short-horizon IO	Medium-horizon IO	Long-horizon IO	Long-horizon minus Short-horizon	Long-horizon minus (Medium-horizon +Short-horizon)	Total IO
$\beta_{12}^{\mathrm{MKT}}$	0.109	1.453	-0.279	-0.387	-1.851	1.318
	[4.66]	[5.25]	[-4.98]	[-5.23]	[-5.93]	[4.64]
$\beta_{24}^{\mathrm{HML}}$	-0.003	-0.117	0.111	0.113	0.238	0.002
	[-0.53]	[-0.97]	[4.26]	[4.29]	[2.04]	[0.01]
β_{3}^{LIQ}	-0.094	-0.528	0.444	0.531	1.119	-0.294
	[-1.83]	[-1.09]	[4.47]	[4.67]	[2.19]	[-0.58]
Size	-0.114	-1.025	0.808	0.924	1.953	-0.394
	[-4.31]	[-4.32]	[12.77]	[20.12]	[8.78]	[-1.32]
B/M	-0.075	-3.683	-1.031	-0.958	2.598	-4.740
	[-3.23]	[-11.19]	[-15.31]	[-13.01]	[7.07]	[-15.31]
$r_{11,-2}$	0.459	2.217	-1.362	-1.833	-4.058	1.438
	[11.20]	[3.61]	[-8.22]	[-10.87]	[-6.60]	[2.13]
Adjusted R^2	0.11	0.11	0.14	0.17	0.14	0.10

Figure 1: Betas vs. Horizon for Size Decile Portfolios

This figure plots the betas of size decile portfolios relative to the value-weighted market portfolio as a function of the return horizon used in the estimation. The size decile and market portfolio returns are monthly returns from the Center of Research in Security Prices (CRSP) stock data base over the period January 1926 to March 2014. For a k-month horizon we compound the monthly decile and market portfolio returns to a k-month horizon and regress the k-month overlapping decile returns on the k-month overlapping market returns.

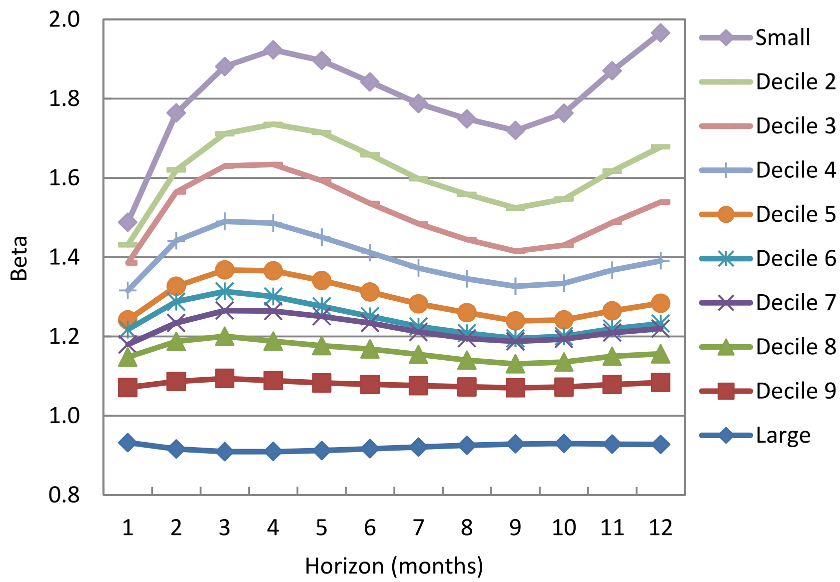


Figure 2: Variance Ratio

Each traded factor (MKT, SMB, HML, and UMD) represents excess return portfolios. For example, MKT is the market return in excess of the risk–free rate; SMB is the return of small firms in excess of big firms. A k-period variance ratio is defined as the ratio of variance of the factor over a k-period horizon and the product of k and the variance at the one-period horizon. $\text{VR}(k) = \text{var}(r^c_{k,t})/[k \cdot \text{var}(r^c_{1,t})]$, where $r^c_{k,t}$ is the continuously compounded excess return for period t over a k-period horizon for traded factors and unexpected liquidity of horizon k for nontraded factor LIQ. For example, $r^{c,\text{MKT}}_{k,t} = \ln[\prod_{\kappa=1}^{k}(1 + r^m_{1,t-k+\kappa})] - \ln[\prod_{\kappa=1}^{k}(1 + r^f_{1,t-k+\kappa})]$. The nontraded liquidity factor LIQ of horizon k in month t is the realized market liquidity level in month t, less its expected value at month $t-k$. To compute the expected value of liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2013, and the expected market liquidity level in month t of horizon k is the k-month-ahead forecasted market liquidity at month $t-k$. The sample period is 1963 through 2013.

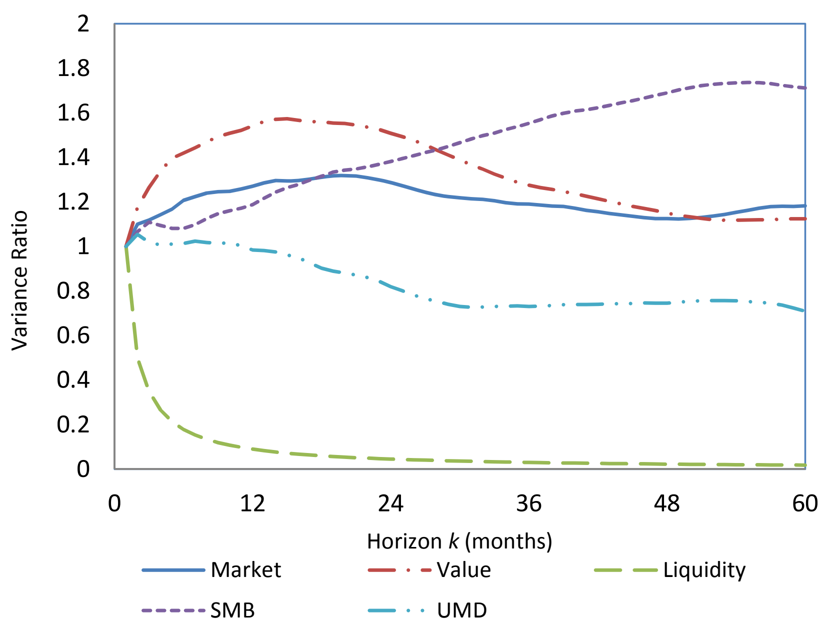


Figure 3: Average Return Spread in Six-Month Intervals

The figure plots the average value-weighted return spread (annualized and in percent) between the top beta decile and the bottom beta decile in each six-month interval against the number of months of returns (k) used in estimating betas. Betas are estimated using overlapping k-month returns in the five years prior to the beginning of each year when portfolios are formed based on various betas. The factors used in estimating betas include Fama–French three factors, UMD, and the Pástor–Stambaugh Liquidity Factor. Penny stocks are excluded and portfolios are value-weighted. Sample period is January 1965 through December 2013.

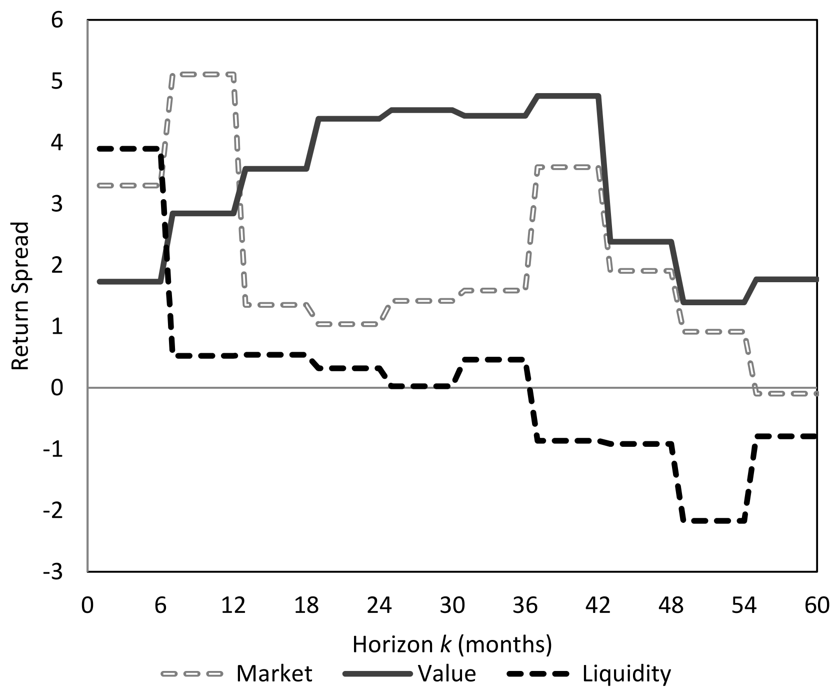
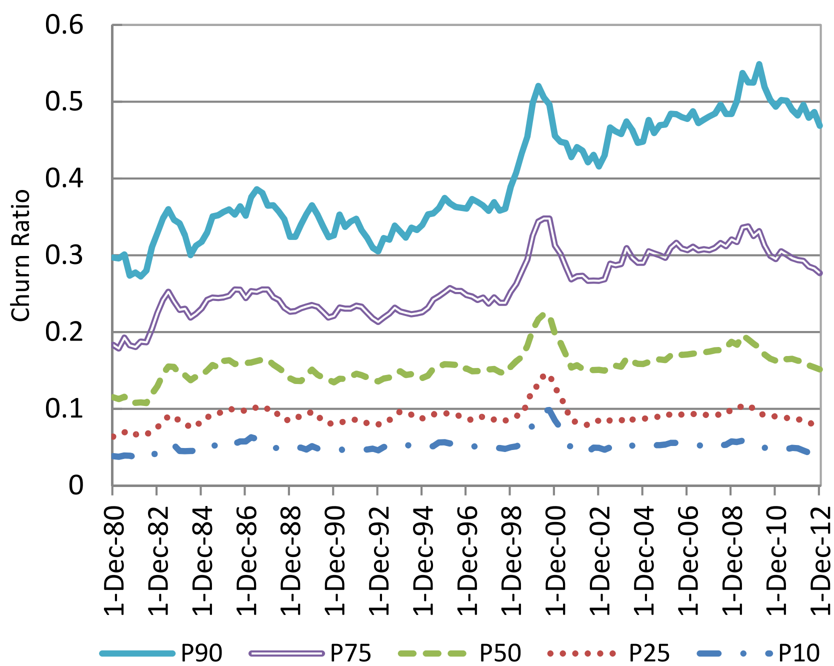


Figure 4: Time Series of Investor Churn Ratio

The figure plots the time series of the cross-sectional distribution statistics of institution churn ratios. For each quarter q and institution j from 1980 to 2012, we calculate the churn ratio

$$\text{CHURN}_{j,q} = \frac{\sum_{i=1}^{n} |\text{Shares}_{i,j,q}\text{P}_{i,q} - \text{Shares}_{i,j,q-1}\text{P}_{i,q-1} - \text{Shares}_{i,j,q-1}\Delta\text{P}_{i,q}|}{\sum_{i=1}^{n}(\text{Shares}_{i,j,q}\text{P}_{i,q} + \text{Shares}_{i,j,q-1}\text{P}_{i,q-1})/2}.$$

where $\text{Shares}_{i,j,q}$ is the number of shares of firm i owned by institution j at the end of quarter q and $P_{i,q}$ is the price of stock i at the end of quarter q. We then calculate $\text{CHURN}_{j,q,4}$ as the average churn ratio in the four quarters $[q-3,q]$ for each j and q. Plotted are the cross-sectional distribution statistics (e.g., P10 denotes the 10^{th} percentile) of $\text{CHURN}_{j,q,4}$ for each quarter from December 1980 to December 2012.



Internet Appendix: Horizon Pricing

Avraham Kamara, Robert A. Korajczyk, Xiaoxia Lou, and Ronnie Sadka

April 6, 2015

Internet Appendix

This appendix contains ancillary results for "Horizon Pricing."

I. Additional Tests

A. Illiquidity

Chordia, Subrahmanyam, and Tong (2014) find that the profitability of a number of asset-pricing anomalies has significantly declined for the subset of liquid stocks. Motivated by their results, we conduct an analysis of horizon pricing for two subsets of stocks sorted by their liquidity. In particular, we calculate the Amihud (2002) liquidity measure for each stock per year (using firms with at least 100 daily observations). "Liquid" ("illiquid") stocks are those with below (above) median Amihud measures in the previous year. We then estimate the cross-sectional regressions, as in Table 4, separately for each liquidity group. The regressors are standardized each period by their sample mean and cross-sectional standard deviation.

Table A1 presents the results. The main conclusion is that the β_{24}^{HML} and β_{3}^{LIQ} are not only priced among illiquid stocks. Therefore, these results are inconsistent with these estimated premia being a result of mispricing in the illiquid segment of the equity market.

B. Returns Across Various Holding Periods

Thus far, we estimate factor exposures over horizons ranging from one month to 61 months, while stocks are held for one month after portfolio formation when estimating risk premia. One might argue that the risk premia for long-horizon investors should be estimated using long-horizon returns for the dependent variable, either in portfolio sorts or Fama–MacBeth regressions. We argue that using short-horizon returns makes sense for several reasons. If a risk premium exists for a given factor using long-horizon betas, it will be present in short-horizon returns and available for both short-horizon and long-horizon investors. The main effect in returns is likely to be the risk premium, rather than the compounding effect of Levhari and Levy (1977). Using short-horizon returns to estimate the risk premium leads to more precise estimates of the risk premium due to the loss of degrees of freedom in using long-horizon returns. Additionally, using long-horizon returns to estimate the risk premium (using individual assets in the cross-sectional regressions) induces a survivorship bias into the risk premium estimates. For example, using 24-month returns to estimate the premium for 24-month beta risk will eliminate any asset that is delisted any time in the 24-month period after the ranking period. In essence, it is impossible to be a truly long-term buy-and-hold investor when assets disappear. The increased precision and reduced survivorship bias led us to choose to estimate risk premia using short-horizon returns for the bulk of our work.

However, as a check on the robustness of these results, we also estimate premia using two alternative methods of computing mean intermediate-horizon returns. The first method follows the Jegadeesh and Titman (1993) approach in calculating monthly returns of longer horizon buy-and-hold portfolios. Specifically, in each month t, we sort the stocks into ten value-weighted decile portfolios based on their previously estimated k-month factor beta, where k=1, 6, 12, 24 for MKT; k=1, 12, 24, 36 for HML; and k=1, 3, 6, 12 for LIQ. We form zero-cost, top-minus-bottom beta decile portfolios and hold them for h months, where

h ranges over all the values of k of each factor. In each month, we close out the positions initiated in month $t - h$. That is, under this trading strategy, each month, we revise the weights of $1/h$ of the securities in each zero-cost, factor/beta portfolio, and carry over the rest of the portfolios from the previous month. As in the previous analyses, betas are computed once a year (at year-end), and the most recent beta is used for portfolio formation.

Table A2, Panel A, shows that the factors MKT and HML continue to behave like intermediate-horizon risk factors, whereas LIQ continues to behave like a short-horizon risk factor. The average returns on portfolios sorted on the six-month MKT beta are significant at the 5% level for holding periods between 1 to 12 months. For the 12-month MKT beta, the returns are significant for a holding period of one month. For beta estimation periods of 24 and 36 months, five of the eight average returns on portfolios sorted on HML betas are significant at the 5% level, while the rest are significant at the 10% level. For LIQ beta portfolios sorted by three- and six-month horizon, seven of the eight average portfolios returns are significant at the 5% level (and the one remainder at the 10% level). The FF4 alphas on the LIQ portfolios are always significant at the 5% level at all holding periods when sorted on betas estimated over 1-, 3- and 6-month periods. Additionally, for every factor and for each value of k, we test for differences in estimated premia across different holding horizons h, and fail to reject they are equal. The driver of the horizon premia is therefore the estimation period k rather than the holding period h.

The results of the second method are reported in Panel B. We estimate cumulative buy-and-hold portfolio returns for each horizon h and annualize return spreads. Portfolios are formed each month, resulting in overlapping return series; standard errors are Newey-West adjusted using $h-1$ lags. The results are generally consistent with those reported in Panel A.

C. Incremental Betas

The analysis above investigates the pricing of the nine betas as distinct variables. In this section we examine the incremental contribution of estimating a beta over a longer horizon rather than over a shorter horizon. For example, rather than studying β_1^{HML}, β_{12}^{HML}, and β_{24}^{HML}, we examine β_1^{HML}, $\beta_{12}^{HML} - \beta_1^{HML}$, and $\beta_{24}^{HML} - \beta_{12}^{HML}$.

Table A3 reports the results using the Fama–MacBeth (1973) cross-sectional analysis in which characteristics are also included as explanatory variables, similar to Table 4. Panel A repeats the cross-sectional analysis of the betas in Columns 11, 12, and 13 of Table 4, but because our objective here is to test the significance of differences in betas, the units in Table A3 are not standardized. Panel B reports the results for the differences in betas. Consistent with the results in Table 4, the contribution of the one-month market beta and the contributions of $\beta_6^{MKT} - \beta_1^{MKT}$, and $\beta_{12}^{MKT} - \beta_6^{MKT}$ are insignificant. In contrast, the contribution of $\beta_{12}^{HML} - \beta_1^{HML}$ is significant at the 10% level and that of $\beta_{24}^{HML} - \beta_{12}^{HML}$ is significantly positive at 5%. Lastly, while β_3^{LIQ} is significantly positively priced at 10% in Panel A, Panel B shows that the incremental contributions of $\beta_3^{LIQ} - \beta_1^{LIQ}$ and $\beta_6^{LIQ} - \beta_3^{LIQ}$ are not significant. Thus, the changes in HML betas from a one-month to a one-year horizon and from a one-year to a two-year horizon have significant explanatory power for the cross section of returns, consistent with our prior results. Additionally, liquidity risk is priced at short horizons, but the changes in betas at longer horizons have no explanatory power for returns.

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Table A1: Stock Illiquidity

The Amihud (2002) measure of stock i in year y, $ILLIQ_{i,y}$, is the average daily ratio of absolute return to the dollar trading volume, averaged over all the trading days in the year where the daily ratio is defined. We require at least 100 valid daily ratios for a stock to be included in the sample. In each month in year y from 1965 to 2013, we estimate value-weighted monthly Fama–MacBeth regressions for groups separated by the Amihud measure in year y-1. Those with an Amihud measure above the median are illiquid stocks, and those below the median are liquid stocks. We exclude penny stocks from our sample. We standardize each independent variable in the entire cross section to a mean of zero and a standard deviation of one. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional regression coefficients, weighted by the inverse of the standard errors of monthly coefficients. Penny stocks are excluded and the sample period is from 1965 to 2013.

	Panel A: Liquid Stocks			Panel B: Illiquid Stocks		
	(1)	(2)	(3)	(1)	(2)	(3)
β_{12}^{MKT}	0.04			0.02		
	[0.89]			[0.73]		
β_{24}^{HML}		0.13			0.07	
		[2.79]			[2.06]	
β_{3}^{LIQ}			0.09			0.05
			[2.14]			[2.06]
Size	-0.10	-0.11	-0.10	-0.06	-0.09	-0.05
	[-1.76]	[-1.91]	[-1.74]	[-0.64]	[-0.95]	[-0.52]
B/M	0.12	0.07	0.10	0.18	0.17	0.18
	[1.76]	[1.09]	[1.57]	[5.07]	[4.92]	[5.16]
$r_{11,-2}$	0.36	0.31	0.33	0.19	0.16	0.19
	[3.59]	[3.17]	[3.37]	[3.81]	[3.15]	[3.80]
Intercept	1.07	1.11	1.07	0.85	0.90	0.85
	[4.63]	[4.69]	[4.66]	[4.09]	[4.32]	[4.12]
Adjusted R^2	0.09	0.09	0.09	0.04	0.03	0.03

Table A2: Varying Holding Period

This table reports results on estimating risk premia using two methods of computing mean intermediate-horizon returns. The first method presented in Panel A follows the Jegadeesh and Titman (1993) approach in calculating monthly returns of longer horizon buy-and-hold portfolios. Specifically, in each month t, we sort the stocks into ten value-weighted decile portfolios based on their previously estimated k-month factor beta, where k=1, 6, 12, 24 for MKT; k=1, 12, 24, 36 for HML; and k=1, 3, 6, 12 for LIQ. To increase power, we also use the portfolios corresponding to the adjacent horizons for horizons greater than one month. For example, to calculate the portfolio return spread of a one-year horizon beta, we use the portfolio returns of 11-, 12-, and 13-month horizons. That is, we average the returns of the three portfolios per month to create a time series of monthly excess returns. We form zero-cost, top-minus-bottom beta decile portfolios and hold them for h months, where h ranges over all the values of k of each factor. In each month, we close out the positions initiated in month t-h. That is, under this trading strategy, each month, we revise the weights of $1/h$ of the securities in each zero-cost, factor/beta portfolio, and carry over the rest of the portfolios from the previous month. Betas are computed once a year (at the previous year-end), and the most recent beta is used for portfolio formation. Reported are the average annualized percentage return spread (with T-statistic in brackets) between the top beta decile and the bottom beta decile for each beta, k and h combination. In Panel B, we estimate cumulative buy-and-hold portfolio returns for each horizon h and annualized return spreads. Portfolios are formed each month, resulting in overlapping return series; standard errors are Newey-West adjusted using h-1 lags. Penny stocks measured at the beginning of each portfolio ranking month are excluded. The sample period is 1965 through 2013.

	β_k^{MKT} Return Spread					β_k^{HML} Return Spread					β_k^{LIQ} Return Spread				Alpha Spread			
Panel A: Jegadeesh-Titman (1993) Approach (Annualized Return Spread)																		
h	k=1	6	12	24		k=1	12	24	36		k=1	3	6	12	k=1	3	6	12
1	0.41	5.68	4.08	1.03	1	2.01	3.56	4.78	4.72	1	2.89	3.84	4.04	0.25	4.82	4.44	4.34	0.84
	[0.16]	[2.75]	[2.13]	[0.55]		[0.67]	[1.64]	[2.27]	[2.39]		[1.46]	[2.10]	[2.24]	[0.16]	[2.36]	[2.35]	[2.35]	[0.51]
6	-0.42	4.37	2.42	-0.17	12	1.92	3.41	4.14	3.25	3	2.64	3.61	4.00	0.70	4.37	3.94	4.14	1.05
	[-0.17]	[2.22]	[1.38]	[-0.10]		[0.72]	[1.70]	[2.13]	[1.78]		[1.35]	[2.00]	[2.24]	[0.44]	[2.16]	[2.12]	[2.25]	[0.64]
12	-0.02	3.95	2.09	-0.33	24	2.24	2.65	3.27	3.31	6	2.89	3.54	3.81	1.14	4.56	3.78	3.92	1.29
	[-0.01]	[2.07]	[1.25]	[-0.19]		[0.89]	[1.46]	[1.87]	[2.01]		[1.49]	[2.02]	[2.16]	[0.73]	[2.28]	[2.09]	[2.16]	[0.80]
24	1.11	3.08	1.10	0.03	36	1.29	2.06	2.98	3.48	12	2.91	3.28	3.66	1.44	4.40	3.50	3.75	1.23
	[0.50]	[1.71]	[0.69]	[0.02]		[0.54]	[1.26]	[1.88]	[2.27]		[1.54]	[1.94]	[2.13]	[0.94]	[2.25]	[2.00]	[2.11]	[0.78]
Panel B: Overlapping Cumulative Return (Annualized Return Spread)																		
h	k= 1	6	12	24		k=1	12	24	36		k=1	3	6	12	k=1	3	6	12
1	0.41	5.68	4.08	1.03	1	2.01	3.56	4.78	4.72	1	2.89	3.84	4.04	0.25	4.82	4.44	4.34	0.84
	[0.16]	[2.75]	[2.13]	[0.55]		[0.67]	[1.64]	[2.27]	[2.39]		[1.46]	[2.10]	[2.24]	[0.16]	[2.32]	[2.35]	[2.36]	[0.49]
6	-0.68	4.64	2.63	-0.54	12	1.25	3.35	4.43	3.59	3	2.67	3.80	3.98	0.61	2.36	4.65	3.39	0.71
	[-0.30]	[2.54]	[1.57]	[-0.29]		[0.43]	[1.49]	[1.97]	[1.81]		[1.62]	[2.50]	[2.63]	[0.40]	[1.27]	[2.77]	[2.17]	[0.40]
12	-1.04	4.23	1.78	-1.20	24	1.53	2.65	3.40	4.13	6	3.25	3.83	4.05	1.16	2.96	5.29	4.59	2.33
	[-0.45]	[2.09]	[1.12]	[-0.61]		[0.55]	[1.24]	[1.29]	[1.82]		[1.90]	[2.46]	[2.61]	[0.74]	[1.80]	[3.10]	[2.46]	[1.31]
24	0.22	3.25	0.16	-1.33	36	0.37	2.23	3.69	4.75	12	3.43	3.59	4.11	1.74	2.03	1.83	1.87	-0.27
	[0.09]	[1.50]	[0.09]	[-0.65]		[0.12]	[0.94]	[1.41]	[1.89]		[1.77]	[1.95]	[2.32]	[1.02]	[1.22]	[0.84]	[0.96]	[-0.15]

Table A3: Fama–MacBeth Regression Results—Incremental Beta

The table reports the results of Fama–MacBeth regressions using nonstandardized independent variables. In each month t, we perform value-weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. For all betas in the regression of month t, the average beta of the decile portfolio that a firm is assigned to based on that beta is used for the firm beta. Independent variables are not standardized. β_1^{MKT} is the market beta estimated using monthly returns in the years $[y-5, y-1]$ for each month t in year y. β_6^{MKT} is the market beta estimated using overlapping six-month cumulative returns. $\beta_6^{MKT} - \beta_1^{MKT}$ is the difference between β_6^{MKT} and β_1^{MKT}. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional coefficients, weighted by the inverse of the standard errors of monthly coefficients. Penny stocks are excluded and the sample period is from 1965 to 2013.

Panel A: Betas (Nonstandardized)			
	(1)	(2)	(3)
β_1^{MKT}	-0.13		
	[-1.56]		
β_6^{MKT}	0.00		
	[0.05]		
β_{12}^{MKT}	0.04		
	[1.05]		
β_1^{HML}		0.02	
		[0.38]	
β_{12}^{HML}		-0.02	
		[-0.91]	
β_{24}^{HML}		0.06	
		[2.92]	
β_1^{LIQ}			[0.09]
			[0.85]
β_3^{LIQ}			[0.15]
			[1.84]
β_6^{LIQ}			[-0.05]
			[-0.98]
Size	-0.06	-0.06	-0.05
	[-2.31]	[-2.35]	[-1.93]
B/M	0.18	0.10	0.16
	[2.24]	[1.41]	[1.90]
$r_{11,-2}$	0.67	0.51	0.54
	[4.01]	[3.22]	[3.29]
Intercept	1.17	1.24	1.07
	[4.21]	[3.66]	[3.19]
Adjusted R^2	0.10	0.10	0.09

Panel B: Differences in Betas			
	(1)	(2)	(3)
β_1^{MKT}	-0.06		
	[-0.60]		
$\beta_6^{MKT} - \beta_1^{MKT}$	0.06		
	[1.34]		
$\beta_{12}^{MKT} - \beta_6^{MKT}$	0.04		
	[1.05]		
β_1^{HML}		0.07	
		[1.05]	
$\beta_{12}^{HML} - \beta_1^{HML}$		0.04	
		[1.78]	
$\beta_{24}^{HML} - \beta_{12}^{HML}$		0.06	
		[2.92]	
β_1^{LIQ}			0.21
			[2.12]
$\beta_3^{LIQ} - \beta_1^{LIQ}$			0.11
			[1.32]
$\beta_6^{LIQ} - \beta_3^{LIQ}$			-0.05
			[-0.98]
Size	-0.06	-0.06	-0.05
	[-2.31]	[-2.35]	[-1.93]
B/M	0.18	0.10	0.16
	[2.24]	[1.41]	[1.90]
$r_{11,-2}$	0.67	0.51	0.54
	[4.01]	[3.22]	[3.29]
Intercept	1.17	1.24	1.07
	[4.21]	[3.66]	[3.19]
Adjusted R^2	0.099	0.102	0.090