Choose Your Betas: *Benchmarking Alternative Equity Index Strategies*

NOËL AMENC, FELIX GOLTZ, AND ASHISH LODH

NOËL AMENC
is a professor of finance at EDHEC Business School, director of the EDHEC Risk Institute, and CEO of ERI Scientific Beta in London, UK.
noel.amenc@edhec-risk.com

FELIX GOLTZ
is the head of applied research at the EDHEC Risk Institute and research director of ERI Scientific Beta in Nice, France.
felix.goltz@edhec-risk.com

ASHISH LODH
is a quantitative analyst at ERI Scientific Beta in Nice, France.
ashish.lodh@edhec-risk.com

There has been increasing interest in so-called alternative equity index strategies or advanced beta strategies, which try to generate outperformance over the standard market indices. These indices are being marketed on the basis of a number of shortcomings of cap-weighted indices, which have been documented to be overly concentrated (see Tabner [2007] and Malevergne et al. [2009]) and to provide poor risk-adjusted returns (see Goltz and Le Sourd [2011] for a literature review). As an alternative to cap-weighted indices, numerous advanced beta equity offerings have been launched that draw on either firm fundamentals or risk–return parameters to construct systematic equity portfolios.

Providers of such indices have widely documented the superior performance of their respective approaches compared with the corresponding cap-weighted indices. In early articles, such performance comparisons have fallen short of accounting for exposures of advanced beta strategies to standard equity risk factors, such as value and small cap (see Arnott, Hsu, and Moore [2005] as an example).[1] Following criticism that underlined the importance of such exposures for explaining part of the outperformance over cap-weighted indices (see, for example, Jun and Malkiel [2007], Kaplan [2008], Blitz and Swinkels [2008], or Amenc, Goltz, and Le Sourd [2009]), factor exposures of advanced beta equity strategies are now widely documented.[2] However, providing an analysis of risk factor exposures does not necessarily allow for a detailed understanding of the construction mechanisms of advanced beta strategies.

Commercially available strategies are bundles of various methodological choices, and performance and risk analyses of such prepackaged indices do not provide for a clear understanding of how the different parts of the methodology influence the overall investment outcome. Moreover, a lack of free access to constituent weights over the period of the track record and incomplete information on the exact construction method of many indices have contributed to the confusion about which mechanisms drive the performance and risks. This confusion has only been reinforced by performance comparison studies that claim to provide a neutral comparison of advanced beta strategies but may contain biases in favor of a particular method (see the comparison proposed in Chow et al. [2011] and Arnott [2011] and the description of biases by Amenc, Goltz, and Martellini [2011a] and Amenc [2011]).

This article proposes to benchmark advanced beta strategies by flexibly combining the results of the different choices for the key steps in strategy construction.[3] In fact, index construction draws on i) a constituent selection that reflects which stocks with which associated characteristics an investor wants to hold, and ii) a weighting scheme that should determine how an investor wants to diversify across the chosen universe of stocks. We construct advanced beta benchmarks by drawing on a variety of weighting schemes (such as minimum volatility with norm constraints, maximum decorrelation, efficient maximum Sharpe ratio, equal weighted, and fundamental weighted) and a variety of stock selections (such as high and low volatility, dividend yield, and market cap).[4] Such benchmarks not only allow for an assessment of a wide range of strategy construction possibilities, but are also a useful tool to assess commercial index strategies by comparing them to benchmarks with similar objectives and constraints.

Our results show that a sole focus on stock selection often allows for improving a given objective only if one is willing to accept a high concentration within the portfolio. For example, to reach the same low volatility level as a diversified minimum-volatility benchmark, a strategy solely based on selecting stocks based on their low volatility and equal weighting them leads to a drastically higher concentration. However, stock selection is a useful tool to define the characteristics that an investor wants to be exposed to, whatever the weighting scheme. In fact, alternative weighting schemes typically lead to significant exposure to equity risk factors, even in the case of diversification schemes that do not have any explicit objective of tilting toward particular risk factors.

Therefore, we test empirically whether biases such as value or small cap that arise when using alternative weighting schemes can be avoided through a suitable stock selection. Our empirical results show, for example, that even though applying various diversification-based weighting schemes in a standard index universe leads to significant small-cap exposure, this exposure either becomes insignificant or reduces by more than 90% when applying the weighting scheme to a selection of the largest stocks by market capitalization within the standard index universe—while still delivering outperformance through the improved stock weighting. In the same way, each of the weighting schemes also leads to a value bias when applied to the standard universe of stocks. When avoiding the stocks with the most pronounced value characteristics in the stock selection, however, any value bias is avoided, and the performance benefits of the alternative weighting schemes are maintained. This evidence contradicts the conventional wisdom that any outperformance of alternative weighting schemes over cap weighting can be fully explained by differences in risk factor exposure.

Finally, to illustrate how the components of commercial equity index strategies can be disentangled, we provide an analysis of popular advanced beta strategies by analyzing the results of some of their methodological components. First, we construct advanced beta benchmarks that allow for assessing several methodological choices of defensive equity strategies, such as the selection of a weighting scheme in a universe of defensive stocks, the combination of a liquidity objective with that of a low-volatility objective, and the selection of constraints within minimum-volatility weighting strategies. Second, we consider various advanced beta benchmarks that draw on a fundamentals-based stock selection. While commercially available indices that use a fundamentals-based stock selection typically combine this selection with a weighting by similar firm fundamentals, there is no reason in principle to restrict the weighting scheme to drawing on the same set of fundamentals. Our results show that selecting stocks by firm fundamentals and using a diversification scheme maintains the outperformance of the fundamentals-based stock selection and improves the relevant diversification objective.[5]

By flexibly drawing on different possible specifications of the two index construction steps (stock selection and weighting scheme), the advanced beta benchmarks we construct disentangle the sources of risk and return. We hope that our analysis helps to improve the understanding of alternative indexation approaches through a transparent assessment of their construction methodologies that goes beyond simply analyzing the overall outcome of the different prepackaged methodologies (for such analyses, see, for example, Amenc, Goltz, and Martellini [2011b], de Carvalho, Lu, and Moulin [2012], Lee [2011], or Melas, Briand, and Urwin [2011]).

The remainder of this article is organized as follows. After reviewing the important issues with commercial advanced beta strategies, which are bundles of methodological choices, we provide an empirical analysis of flexibly designed advanced beta benchmarks, which disentangles the weighting scheme decision from the stock selection decision. We then create benchmarks for the assessment of index construction steps used in popular advanced beta products, including low-volatility strategies and fundamental equity indexation strategies. The final section presents our concluding remarks.

ISSUES WITH ADVANCED BETA EQUITY STRATEGIES: CONCEPTUAL CONSIDERATIONS

An Overview of Existing Advanced Beta Equity Indices

One can distinguish two different angles of improvement over cap-weighted equity indices. A first set of approaches aims for a better representation of the economy. Wood and Evans [2003] introduced the concept of fundamental equity indexation (see also Arnott, Hsu, and Moore [2005]). Such indices maintain key characteristics of standard cap-weighted indices in order to facilitate their adoption as substitutes for the latter. For example, fundamental equity indexation emphasizes low turnover and is based on an intuitive weighting scheme that works in a similar fashion to cap weighting. Such indices simply replace the market cap by a different measure of firm size based on variables such as profits, book–value, and revenue. A second set of approaches aims at providing an approximation of the risk–return optimal portfolio (tangency portfolio) through diversification techniques. Such approaches draw on the recognition that cap-weighted indices have been shown to be poor proxies of the tangency portfolio (see Ferson, Kandel, and Stambaugh [1987], as well as the literature review by Goltz and Le Sourd [2010] and the references therein).

A special case of an index that aims at better diversification is the equal-weighted index. In the professional and academic literature, this index is perceived not only as an extremely robust proxy of the tangency portfolio to which it corresponds under very restrictive assumptions (of identical risk and return parameters for all stocks[6]), but also, most importantly, as the main example of a series of ad hoc approaches. These approaches do not aim at diversification—in the sense of exploiting correlation properties across stocks, which is at the heart of modern portfolio theory—but rather at ad hoc deconcentration (i.e., spreading out the monetary amounts that are invested over the largest possible effective number of stocks[7]). This idea of deconcentration has lead to approaches such as equal risk contribution, which are based on ad hoc deconcentration on a risk basis rather than ad hoc deconcentration on a dollar basis (see Maillard, Roncalli, and Teiletche [2010]). However, the underlying philosophy of such approaches remains one of ad hoc deconcentration rather than diversification.[8]

Before turning to an overview of how different advanced beta equity strategies are constructed in practice, it is of interest to illustrate the exposure to equity risk factors that the commercially available advanced beta equity indices lead to. For brevity, we focus on a selection of four non-cap-weighted indices in the U.S. universe over a relatively short horizon for which data for all of these indices are available. Exhibit 1 presents the factor exposures with respect to four factors: the market factor, the size factor (high minus low market-cap stocks), the volatility factor (return of high-volatility stocks minus return of low-volatility stocks) and a value factor, or more precisely, a dividend-yield factor (return of high-dividend-yield stocks minus return of low-dividend-yield stocks). The betas, which are significant at 1% level, are highlighted in bold. It appears that all indices have highly significant exposures to at least one of the standard equity risk factors. Moreover, when considering the percentage of variability in excess returns over

EXHIBIT 1

Factor Exposures of Commercial Advanced Beta Equity Strategies

The exhibit shows the excess (over the S&P 500 Index) risk factor exposures of the S&P 500 Equal Weighted Index, MSCI USA Minimum Volatility Index, FTSE EDHEC Risk Efficient US Index, and FTSE RAFI US 1000 Index. Weekly total return data from January 3, 2003–December 31, 2010, are used for the analysis, and values significant at the 1% level are highlighted in bold. The following regression is run for each index by using the excess weekly returns (over the cap-weighted index) of the indices over the period of analysis. In the regression below, the risk exposures are the excess risk exposures over the cap-weighted S&P 500 Index.

$$R_P - R_{CW} = \alpha + \beta_1 \cdot (R_{CW} - R_{rf}) + \beta_2 \cdot Size + \beta_3 \cdot Vol + \beta_4 \cdot DY$$

	S&P EW	MSCI Minimum Volatility	FTSE EDHEC-Risk Efficient	FTSE RAFI
Alpha	0.02%	0.02%	0.04%	0.03%
Market	0.99%	**–7.80%**	–3.41%	–1.36%
Size (Big-Small)	**–31.82%**	**–8.05%**	**–23.60%**	**–12.35%**
Volatility (High-Low)	0.17%	**–13.52%**	–0.20%	5.42%
Dividend Yield (High-Low)	–1.52%	**13.94%**	–1.75%	**14.68%**
R^2	86.74%	63.45%	53.07%	67.71%

the cap-weighted index that is explained by differences in risk factor exposures (the R^2 in Exhibit 1), it is clear that such risk factor exposures explain an important part, but not all, of the performance of advanced beta strategies, with the R^2 ranging from slightly more than 50% to almost 90%.

While such risk factor exposures may or may not be appropriate for a given investor, it is important to understand which underlying mechanisms explain such differences in risk exposure. In fact, in order to improve upon standard cap-weighted indices, commercially available advanced beta strategy indices deviate from the standard market-cap index in terms of stock selection, stock weighting, or both of these index construction steps. Exhibit 2 indicates the choices of various commercial advanced beta indices.

It is apparent from Exhibit 2 that a large variety of approaches exist to combine selection decisions and weighting scheme decisions. However, it is not entirely clear whether stock selection or weighting would be the most appropriate way to reach a given investment objective. For example, the FTSE GWA index series and the FTSE RAFI index series weight stocks by fundamental characteristics, including cash flow and book value, but while the FTSE RAFI index series also uses these fundamentals to select stocks, the FTSE GWA index series does not modify the stock selection of the corresponding cap-weighted index.

Similar differences exist across providers of risk-based indices. The MSCI risk-weighted index reweights stocks in the standard index universe by the inverse of volatility, while the Russell Defensive index instead selects stocks based on their low volatility[9] and weights them by their market cap. The S&P 500 Low Volatility Index adopts yet another approach by selecting stocks based on low volatility and weighting them by the inverse of their volatility. Little evidence or discussion has been provided on whether using the weighting, stock selection, or both index construction steps would be the most appropriate way of reaching an objective.

Moreover, some indices have a stock selection and weighting scheme that each aims at a different objective. Such indices generally do not provide any justification for having chosen weighting or selection as the means of pursuing the respective objective. For example, the S&P GIVI Index selects stocks based on low risk and weights them based on their intrinsic value.[10] It appears natural to ask whether this is preferable to instead selecting stocks by their intrinsic value and weighting them inversely to their risk. Typical information material of index providers does not provide any analysis of such alternatives. Rather, it appears as if the ad hoc index construction approaches in current offerings are driven more by marketing innovations than by a cogent and justified decision framework for each index construction step.

Conceptual Distinctions between Diversification-Based Weighting Schemes and Stock Selection Strategies

The examples on current index offerings show that when trying to attain an investment objective, one can use a stock selection decision or a weighting scheme to obtain the declared objective. Among possible weighting schemes, we have made a distinction between two types of weighting schemes: *diversification-based* weighting schemes consider not only the standalone characteristics of stocks, but also the interaction that arises between stocks when they are combined in a portfolio to attain an objective; *characteristics-based* weighting schemes simply weight stocks proportionally or inversely proportionally to a stock-level characteristic, such as volatility or revenues. In ignoring the dependence structure across stocks, characteristics-based weighting is somewhat similar to stock selection.[11]

In the remainder of this article, we focus on the distinction between a diversification-based weighting scheme and stock selection. We consider only a limited number of examples of characteristics-based weighting when it is appropriate to illustrate issues with popular existing index offerings. Before providing an empirical assessment of various stock selections and diversification-based weighting schemes and combinations thereof, it is useful to distinguish the two concepts from a conceptual perspective.

The stock selection step in index construction is a way of defining where the strategy will be invested in terms of stock-level characteristics. Stock selection by construction takes into account only the standalone properties of stocks and is unable to account for the interaction effects that influence portfolio properties when combining these stocks. Therefore, stock selection from a conceptual perspective is a limited tool when it comes to influencing portfolio risk and return properties. Stock

EXHIBIT 2
An Overview of Stock Selection and Weighting Decisions of Some Alternative Equity Indices

Index	Stock Selection	Stock Weighting
Indices that change only the selection compared to standard index		
Broad Dividend Achievers[1]	Positive dividend growth	Market capitalization (or free float)
MSCI High Dividend Yield[2]	High dividend yield, positive dividend-per-share growth, low dividend payout ratio	
FTSE Active Beta Momentum and Value[3]	High price momentum, high book value-to-price ratio, high sales-to-price ratio, high cash flow-to-a price ratio	
Russell High Dividend Yield[4]	Positive free cash flow, positive return on equity, positive forecasted earnings growth, high price momentum, low debt-to-equity ratio, low EPS variability, high dividend yield, high dividend growth	
Russell Defensive[5]	Low leverage, high return on assets, low earnings variability, low total return volatility	
Indices that change only the weighting		
FTSE GWA[6]	Index universe remains the same as the parent market index	Net income, cash flow, book value
MSCI Value Weighted[7]		Sales, earnings, cash earnings, book value
MSCI Risk Weighted[8]		Inverse of historical variance
MSCI Minimum Volatility[9]		Volatility minimization
S&P 500 Equal-weighted[10]		Equal weighted
FTSE EDHEC-Risk Efficient[11]		Sharpe ratio maximization
FTSE TOBAM Max. Diversified[12]		Maximize diversification ratio
Lyxor SmartIX ERC[19]		Set risk contribution of constituents equal
Indices that change both selection and weighting		
Dow Jones Select Dividend[13]	Positive dividend growth, low dividend payout ratio	Dividend
FTSE RAFI[14]	High sales, high cash flow, high book value, high dividend	Sales, cash flow, book value, dividend
Intellidex[15]	High price momentum, earnings momentum, quality, value, management action	Equal weighted
S&P GIVI[16]	Low market beta	Intrinsic value
S&P 500 High Beta[17]	High market beta	Market beta
S&P 500 Low Volatility[18]	Low volatility	Inverse of volatility

[1]http://www.indxis.com/USBroad.html, [2]http://www.msci.com/products/indices/strategy/risk_premia/hdy/,
[3]http://www.ftse.com/Indices/FTSE_ActiveBeta_Index_Series/index.jsp,
[4]http://www.russell.com/indexes/data/dividend/russell-high-dividend-yield-indexes.asp, [5]http://www.russell.com/indexes/data/stability/russell-stability-indexes.asp,
[6]http://www.ftse.com/Indices/FTSE_GWA_Index_Series/index.jsp, [7]http://www.msci.com/products/indices/strategy/risk_premia/value_weighted/,
[8]http://www.msci.com/products/indices/strategy/risk_premia/risk_weighted/, [9]http://www.msci.com/products/indices/strategy/risk_premia/minimum_volatility/,
[10]http://www.standardandpoors.com/indices/sp-500-equal-weight-index/en/us/?indexId=spusa-500-usdew--p-us-l--,
[11]http://www.ftse.com/Indices/FTSE_EDHEC-Risk_Efficient_Index_Series/index.jsp,
[12]http://www.ftse.com/Indices/FTSE_TOBAM_Maximum_Diversification_Index_Series/index.jsp,
[13]https://www.djindexes.com/dividend/,[14]http://www.ftse.com/Indices/FTSE_RAFI_Index_Series/index.jsp,[15]https://indices.nyx.com/fr/directory/intellidex,
[16]http://www.standardandpoors.com/indices/sp-givi-global/en/us/?indexId=sp-givi-global,
[17]http://www.standardandpoors.com/indices/sp-500-high-beta/en/us/?indexId=spusa-500-usdw-hbp-us-l--,
[18]http://www.standardandpoors.com/indices/sp-500-low-volatility/en/us/?indexId=spusa-500-usdw-lop-us-l--
[19]http://www.ftse.com/Indices/Lyxor%20SmartIX%20ERC%20Equity%20Indices/index.jsp

selection, however, is an explicit and transparent way of tilting a portfolio toward desired stock characteristics or risk factor exposures. Indeed, tilting a portfolio toward certain stock characteristics, such as value or low-volatility stocks, can be achieved without taking into account the dependence structure across stocks; such tilts of a portfolio are simple weighted averages of individual stock characteristics. When aiming at a diversification objective in the sense of an attractive risk–reward trade-off at the portfolio level, stock selection is, however, at worst unsuitable and at best insufficient to achieve such objectives due to the ignorance of the interaction effects that arise when combining stocks.

A diversification scheme, on the other hand, explicitly aims at achieving a risk–reward objective by taking into account how different stocks interact when combined in a portfolio. Importantly, such diversified weighting schemes exploit the phenomenon that portfolio risk is not simply the weighted average risk of its constituents. However, such weighting schemes are not immune to simple factor tilts. For example, when trying to obtain a low-volatility objective only through a weighting decision (volatility minimization), one does not necessarily take into account relevant standalone characteristics of stocks and, as a consequence, may be exposed to unwanted risks stemming from the characteristics of stocks that are favored by the diversification approach, such as sector or style characteristics.[12] A key difference between weighting and stock selection is that the factor tilts resulting from the former are often much more implicit.

Given that selection and diversification come with different advantages and drawbacks, rather than seeing selection and diversification as competing ways of reaching an objective, one could usefully combine them to contribute toward a given objective. For example, to lower volatility, one could select low-volatility stocks and weight them by using a minimum-volatility approach. Another possibility is to combine selection and diversification in the sense that each construction step deals with a different investment objective. In fact, one can consider stock selection as the natural tool to influence which type of characteristics the universe is associated with, and it can thus be used to control some of the implicit risk factor tilts resulting from a diversification-based weighting scheme. For example, stock selection may be a useful tool to correct the risk factor tilts resulting from minimum-volatility weighting by excluding stocks with undesired style characteristics. Such a use of stock selection to control risk factor exposures also avoids using a large number of constraints in the minimum-volatility optimization, which in practice may render the optimization process less effective.

To be able to evaluate the merit of the choices made at each step of the index construction process, a clearly defined objective is required. In this respect, it is surprising to observe that index providers often do not provide an unequivocal definition of the overall objective a given strategy index is supposed to achieve. For example, providers of fundamentals-based equity indices have argued that such indices provide a better mirror of the economy than cap-weighted indices (see Arnott, Sauter, and Siegel [2007]). However, whether such indices attain their objective of improving economic representativeness cannot be measured because it has not been *formally* defined.

In contrast to such a lack of clarity that is sometimes present with ad hoc weighting schemes, a weighting scheme that is based on a diversification technique will necessarily come with an explicit objective, and our approach allows for analyzing how well a given weighting scheme reaches its objective across different stock selections. Thus, in the next section, we will test whether various diversification-based weighting schemes reliably attain their objective across different stock selections, relative to both the broad cap-weighted index and the cap-weighted portfolio within the corresponding stock selection.

DISENTANGLING STOCK SELECTION AND DIVERSIFICATION: EMPIRICAL ILLUSTRATIONS

This section goes beyond the examples of commercial indices discussed above and analyzes methodological choices in advanced beta strategies through the construction of advanced beta benchmarks, which flexibly draw on a large set of possible methodological choices for stock selection and diversification-based weighting. We will first assess the relative merits of selection and diversification to achieve an overall risk–return trade objective. We then assess benchmarks that draw on diversification-based weighting while using stock selection to decide on exposure to risk factors.

Attaining a Risk–Return Objective through Diversification or Stock Selection

In the analysis below, we consider three diversification schemes and assess how well they attain their respective objectives.[13] In particular, we consider the following three diversification approaches:[14]

i. *Minimum-volatility weighting with norm constraints* (GMV-NC): We minimize overall portfolio volatility subject to a limit on portfolio concentration, which is also known as a norm constraint. It is, in fact, common to impose constraints on minimum-volatility portfolios; in the absence of constraints, most of the risk reduction of such portfolios will come from concentrating in stocks with low volatility, as opposed to exploiting correlation effects across stocks in the universe. A standard answer to this problem is to use upper and lower bounds on individual stock weights to reduce such concentration. However, such rigid weight constraints require arbitrary decisions of specifying these constraints for each stock, and the portfolio outcome may be negatively affected by overly restrictive constraints.

   DeMiguel et al. [2009] propose norm constraints as a more flexible alternative to rigid weight constraints. Such norm constraints limit an overall measure of portfolio concentration rather than imposing constraints on the weight for each individual stock. In particular, we impose a lower bound on the *effective number of stocks*. The effective number of stocks is defined as the reciprocal of the Herfindahl Index, a standard measure of portfolio concentration.[15] For a given test portfolio, it indicates the number of stocks that—when equal weighting them—would lead to the same amount of concentration (same Herfindahl Index). To evaluate whether the minimum-volatility strategy attains its objective, we look at the reduction in volatility over the relevant reference index.

ii. *Efficient maximum-Sharpe ratio weighting* (efficient MSR): This approach consists of a formal Sharpe ratio maximization, whereby expected returns are estimated indirectly by assuming that they are proportional to the median downside risk of the risk group a stock belongs to (see Amenc et al. [2011] for a detailed description of the methodology). The assumption of a positive link between risk and return implies that low-risk stocks are penalized by assuming low expected returns. In that sense, this weighting scheme constitutes an alternative to the use of norm constraints within a minimum–variance portfolio. To assess whether the approach attains its objective, we consider whether it consistently improves the Sharpe ratio relative to the reference index.

iii. *Maximum-decorrelation weighting* (MDC): The maximum-decorrelation approach is yet another alternative to using norm constraints within a volatility minimization. The idea is to combine stocks so as to exploit the risk-reduction effect stemming from low correlations rather than reducing risk by concentrating in low-volatility stocks. This weighting scheme corresponds to a minimization of portfolio volatility under the assumption that individual volatilities are identical across stocks (see Christoffersen et al. [2010]). To assess whether the approach attains its objective, we look at a measure of concentration that takes into account correlations. In particular, Goetzmann, Li, and Rouwenhorst (GLR [2005]) use the ratio of the variance of the portfolio returns to the weighted average variance of individual stock returns to take into account not only the distribution of weights in the portfolio but also the correlation properties. More specifically, a portfolio that concentrates weights in assets with high correlation will tend to have a portfolio risk that is high relative to the average stand alone risk of each of its constituents. Thus, it will have a high *GLR measure*.

To assess the results obtained when using these approaches, we will first analyze diversification schemes and stock selection as competing principles to reach an objective. Second, we will consider combining stock selection with diversification-based weighting in a mutually reinforcing manner. For our illustrations, we consider the S&P 500 universe of stocks over a long horizon period from January 1959 to December 2010.

Diversification and selection as competing principles for attaining an objective. We consider the following stock selection approaches as alternatives to diversification-based weighting: As an alternative to minimizing volatility, we select the stocks that have the lowest volatility over the calibration period. As an alternative to efficient maximum Sharpe ratio weighting, we select stocks based on their expected standalone Sharpe ratio, whereby the expected stock returns and volatility are estimated in the same manner as in the diversification-based weighting strategy. As an alternative to the maximum decorrelation approach, we perform a stock selection based on a stock's average correlation with other stocks in the universe.[16] We then assess how well these advanced beta benchmarks[17] attain their respective objectives. In stock selection strategies, we equal weight stocks in order to make sure that no additional information is taken into account. We also set the number of stocks included in the stock selection strategy equal to the effective number of stocks in the diversified portfolios to compare the results at an equal level of portfolio concentration.

The risk and performance statistics for the resulting portfolios are shown in Exhibit 3. The table first shows basic performance and deconcentration measures and then focuses on the measures that relate to the three objectives of our advanced beta benchmarks—volatility, Sharpe ratio, and the GLR measure of correlation-adjusted concentration. The results suggest that the stock selection approaches cannot match the portfolio diversification techniques in terms of attaining the stated objective. Minimum-volatility weighting (denoted GMV NC) achieves 6% lower volatility than low-volatility stock selection. The efficient maximum-Sharpe ratio (Efficient MSR) weighting scheme achieves a Sharpe ratio that is 16% higher than that obtained by selecting the stocks with the highest Sharpe ratio. Likewise, the maximum decorrelation (MDC) approach leads to a Goetzmann-Li-Rouwenhorst (GLR) concentration measure that is 4% lower that obtained through low-correlation stock selection.

Although it may not be surprising that diversification techniques indeed add value in the portfolio construction process, these results cast some doubt on the common practice of relying on stock selection to obtain an objective. Indeed, even though simple stock selection techniques may be preferred by practitioners due to their familiarity with such approaches, our results suggest that diversification approaches, by exploiting the interaction effects that arise when combining stocks in

EXHIBIT 3

Attainment of Objective through Diversification vs. through Stock selection at Identical Concentration Levels

The exhibit presents the performance statistics and deconcentration measures of three pairs each of diversified and stock selection-based indices constructed to achieve the following objectives: low volatility, high Sharpe ratio, and high decorrelation. All statistics are annualized, and performance ratios that involve the average returns are based on the geometric average. The improvement in respective objective refers to the relative decrease in volatility attained by the GMV-NC portfolio, the relative increase in Sharpe ratio attained by the efficient MSR portfolio, and the relative decrease in the GLR concentration measure attained by the MDC portfolio. Weekly total return data from January 2, 1959–December 31, 2010, is used for the analysis.

		Volatility Objective		Sharpe Ratio Objective		Decorrelation Objective	
	Cap Weight	GMV NC	Low Vol Selection	Efficient MSR	High SR Selection	MDC	Low Corr Selection
Ann Returns	9.60%	11.97%	11.95%	11.87%	12,59%	12,06%	12,27%
Tracking Error	0.00%	6.41%	5.95%	5.64%	5,25%	5,96%	5,95%
Effective Number	95.3	166.7	167.0	137.2	137,0	177,9	178,0
Ann Std Dev	15.46%	12.40%	13.19%	12.81%	16,35%	15,19%	14,71%
Sharpe Ratio	0.27	0.53	0.50	0.51	0,44	0,44	0,47
GLR Concentration Measure	0.245	0.155	0.229	0.157	0,231	0,139	0,145
Improvement of Objective through Diversification	–	6%		16%		4%	

a portfolio, lead to a better attainment of objectives at an equal level of concentration.

Exhibit 3 shows that, at an equal level of concentration, the low-volatility benchmark constructed using only security selection has a volatility of 13.19%, compared with a volatility of only 12.40% obtained by a minimum-volatility-weighted benchmark that takes into account not only the standalone risk characteristics of stocks but also the dependence structure of stock returns. Exhibit 4 provides a slightly different analysis to assess which level of concentration is needed to reach the same volatility as the minimum-volatility approach when relying only on stock selection.

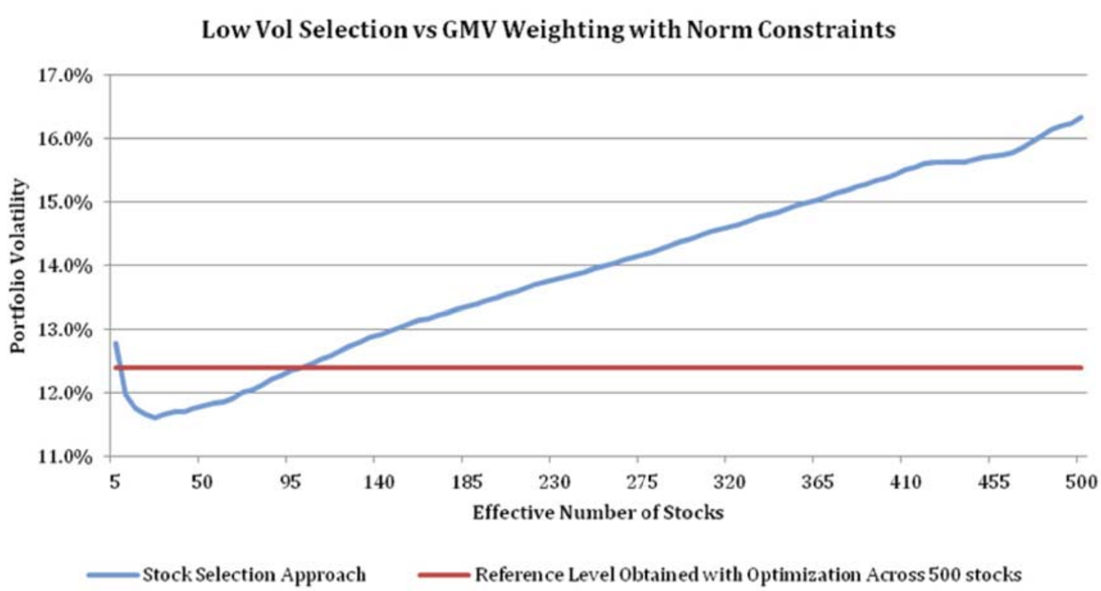
The graph shows that one must concentrate the portfolio to the 95 least volatile stocks in the S&P 500 universe to reach the 12.40% volatility level of the minimum volatility strategy, thus leading to a high level of concentration. The fact that the norm-constrained minimum volatility approach reaches the same level of volatility with an effective number of stocks that is much higher means that exploiting the covariance between the stocks is indeed useful in reducing volatility of the diversification-based portfolio. At concentrations as low as 25–30 stocks, the portfolio based solely on low volatility selection delivers annual volatility that is even lower than that of the minimum volatility portfolio. However, if one further reduces the number of stocks, volatility increases again due to the extremely high concentration.[18]

Diversification and selection as reinforcing elements to attain an objective. The results show that a diversification scheme—by accounting not only for the standalone characteristics, but also for the dependence structure across stocks—is able to reach an overall risk–reward objective more effectively and at lower levels of portfolio concentration than stock selection. It may, however, be natural to ask whether combining a selection decision with a diversification-based weighting

EXHIBIT 4

Concentration Problem When Focusing on Stock Selection to Attain Objective: The Case of Low Volatility

The plot shows, in the curve, the portfolio volatility as a function of stock concentration for a low-volatility portfolio, and the straight line represents the benchmark volatility (i.e., the volatility of the minimum volatility portfolio). Weekly total return data from January 2, 1959–December 31, 2010, are used for the analysis. Note that minimum-volatility optimization across 500 stocks leads to an effective number of stocks equal to 166.7.



Note: For a color version of this exhibit, please visit The Journal of Portfolio Management *website at www.iijournals.com/jpm.*

scheme could lead to even better results. To illustrate this issue, we report results for an advanced beta benchmark that combines a selection of one-third of the stocks with the lowest volatility with minimum-volatility weighting and compares this to the case where only selection or only weighting is used to reduce volatility. Exhibit 5 reports the results.

The results reported in Exhibit 5 show that the volatility reduction of the combined approach of minimizing volatility among low-volatility stocks leads to much lower volatility than a sole stock selection, suggesting that even within a low-volatility stock universe, one can effectively exploit the volatility and correlation properties of stocks to reweight them in order to further lower volatility. Such an approach of combining selection and weighting also allows for reducing volatility compared with the minimum-volatility benchmark based on the standard universe, at the cost of increasing concentration.

Stock Selection as an Explicit Choice of Risk Factor Exposure When Using an Alternative Weighting Scheme

While diversification approaches may be a superior alternative, or at least a very important complement, to pure stock selection approaches when it comes to reaching a risk–return objective, it is clear that any deviation from the standard cap-weighting approach will potentially lead to exposures to equity risk factors that are different from the cap-weighted references. It is therefore sometimes argued that such strategies simply consist of style tilts toward small cap, value, and low-volatility stocks (see Scherer [2011] or Chow et al. [2011]). In this subsection, we assess such factor tilts for the diversified strategies discussed above, and we consider how stock selection can help to control the risk factor exposure.[19]

Exclusion of value stocks to avoid the value tilt of alternative weighting schemes. Diversification schemes applied to the entire universe may end up overweighting value stocks, and such exposure has indeed been widely documented for a variety of diversification-based strategies. However, the minimum-volatility, efficient maximum-Sharpe ratio, and maximum-decorrelation approaches discussed above do not actually consider any valuation characteristics in their construction. The fact that they end up with a value tilt is potentially unwanted collateral of the weighting scheme. Changing the selection of stocks should be a natural way to control such unwanted exposures. For example, an investor could apply the diversification scheme to a stock selection of growth stocks. In that case, it is difficult to imagine that the diversification-based weighting scheme would introduce a value exposure.

We assess whether stock selection effectively allows cancelling the value bias through the construction of advanced beta benchmarks that exclude a certain per-

Exhibit 5
Volatility Reduction Achieved by Combining Minimum-Volatility Weighting and Low-Volatility Stock Selection

The exhibit summarizes the objective attained by the portfolios based on the one-third lowest-volatility S&P 500 stocks. Cap-weighted, equal-weighted, and minimum-volatility portfolios are compared against the broad S&P 500 index. All statistics are annualized, and performance ratios that involve the average returns are based on the geometric average. Weekly total return data from January 2, 1959–December 31, 2010, are used for the analysis.

	All stocks	Low-Volatility Stocks	
	GMV-NC	EW	GMV-NC
	Diversification only	Selection only	Selection and Weighting
Avg excess returns over cap-weighted S&P 500	2.37%	2.14%	2.16%
% Reduction in volatility relative to cap-weighted S&P 500	19.8%	15.8%	26.6%

centage of stocks in the standard universe to avoid value exposure. More specifically, we exclude the highest-dividend-yield stocks and vary the percentage of excluded stocks from only 5% of the stocks, then 10%, 15%, and so forth. We assess at each step whether an alternative weighting scheme leads to a value tilt. For each weighting scheme, we retain the stock selection with the lowest exclusion rate among all selections for which the value exposure is clearly insignificant (*t*-statistic of less than 0.5). Exhibit 6 reports basic risk and return properties of the resulting advanced beta benchmarks, as well as the attainment of the respective diversification objective.

It is clear from these results that all diversification-based weighting schemes allow for improving their respective objective compared with the broad cap-weighted index, even when they are applied to a subuniverse of stocks. In particular, the minimum-volatility approach reduces volatility by 12.6% over the cap-weighted S&P 500, the efficient MSR strategy more than doubles the Sharpe ratio, and the maximum-decorrelation approach reduces the correlation-adjusted concentration measure (GLR measure) by more than 40%. These results are obtained despite the fact that by excluding stocks with the most pronounced value characteristics, the relevant subuniverses are considerably smaller than the full index universe, making up from 45% to 85% of the S&P 500 stocks. It should also be noted that, even though these strategies avoid any significant value exposure, they all have positive excess returns over the cap-weighted S&P 500 index.

Extension to general factor tilts. To assess more generally how stock selection can help to define factor exposures irrespective of the weighting scheme, we divide the S&P 500 universe by volatility, size, and dividend yield sort. Stocks are sorted on the relevant variable and are then divided into three equal groups, ranging from low to high. Exhibit 7 shows the three stock selection approaches and the resulting portfolios.

We construct the same diversified portfolios as in the previous subsection for each of these nine subuniverses and examine the factor tilts in Exhibit 8. Exhibit 8 also shows the factor tilts for the diversified portfolios based on the full S&P 500 universe, which correspond to the portfolios analyzed above. We will first concentrate on an assessment of factor exposures, before turning to the assessment of how well the diversification-based weighting schemes attain their respective objective in Exhibit 9. The objective here is simply to analyze factor exposures. Such exposures could be of interest to investors if, for example, they have a view on the returns to different categories of stocks (such as growth and value), at the same time as having a diversification objective. For example, an investor looking

EXHIBIT 6
Excluding Value Stocks When Using Alternative Weighting Schemes

The exhibit summarizes the performance of diversified strategies based on low-dividend-yield stock selection. The fraction of stocks included, as shown in the first row, is determined separately for each strategy so as to bring the ex post value exposure (from July 5, 1963–December 31,2010) close to zero. We varied the percentage of excluded stocks from 5% to 50% in increments of 5% and selected the lowest exclusion rate that allows for achieving a *t*-statistic (using Newey West [1987] standard errors) for the value exposure coefficient of less than 0.5. norm-constrained minimum-volatility, efficient maximum-Sharpe ratio, and maximum-decorrelation portfolios are compared against the broad S&P 500 index. All statistics (except value exposure and corresponding *t*-statistics) are annualized, and performance ratios that involve the average returns are based on the geometric average. Weekly total return data from January 2, 1959–December 31, 2010, are used for the analysis. The improvement in respective objective refers to the relative decrease in volatility attained by the GMV-NC portfolio, the relative increase in Sharpe ratio attained by the efficient MSR portfolio, the relative decrease in the GLR concentration measure attained by the MDC portfolio.

	Portfolios are based on excluding the relevant proportion of high-dividend-yield S&P 500 stocks so as to render their value tilt insignificant		
	GMV-NC	**Efficient MSR**	**MDC**
% of stocks included	45%	50%	85%
Effective number of stocks	75	74	152
Annualized returns	13,89%	13,12%	12,61%
TE with S&P 500	6,10%	5,33%	5,78%
Volatility	13.51%	13.69%	15.03%
% Improvement in respective objective over CW S&P 500	12.6%	107.1%	40.9%
Value exposure	0.76%	0.16%	0.39%
t-statistic	0.46	0.11	0.25

for a low-volatility portfolio while having a positive view on the performance of growth stocks, may wish to assess whether constructing a minimum-volatility portfolio of growth stocks will effectively maintain exposure to the growth factor.

The table above clearly shows that certain exposures of diversification-based strategies in the full universe can be corrected by a suitable stock selection decision. For example, Panel A of Exhibit 8 shows that the GMV portfolio based on the high-volatility stocks' subuniverse results in positive volatility factor exposure (+15.21%) and hence exposure to high-volatility stocks, whereas the volatility minimization applied to the entire universe results in a volatility exposure of –4.94% and hence a tilt toward low-volatility stocks. Similarly, the small size exposure can be controlled by opting for selecting the largest capitalization stocks in the S&P 500 universe prior to the optimization.

Although all weighting schemes lead to significant exposure to the small-cap factor when applied to the entire universe of S&P 500 stocks, when applying the weighting to the selection of the largest market-cap stocks within this universe, the small-cap tilt becomes insignificant or reduces by at least 90%. Such a selection of the largest-cap stocks before applying an alternative weighting scheme is quite useful; it not only resolves the statistical small-cap tilt, but also may help to resolve any liquidity issues and make the diversification-based strategies easier to implement.

The results in Exhibit 8 also generalize the previous finding that stock selection decisions prior to weighting decisions are useful to control value exposure. In fact, while all diversified strategies yield to a positive value tilt when applied to the entire index universe, when applying these diversification strategies to a selection of growth stocks (low-dividend-yield stocks), a growth tilt results. Another interesting example of how stock selection allows explicitly defining risk factor exposures is the low-volatility maximum-decorrelation portfolio. While the maximum-decorrelation weighting approach has no tendency to tilt toward low-volatility stocks, when applying it to a stock selection of low-volatility stocks, one obtains such an exposure. Hence, applying stock selection along with diversification can be seen as a lever to fine-tune the exposure (as desired by the investor) while reaping the benefits of diversification.

While stock selection decisions help to obtain the desired factor exposures or at least control some of the implicit factor tilts of weighting schemes, an interesting question is whether the stock selection will prevent the weighting scheme from reaching its declared objective. In Exhibit 9, we assess whether each diversification approach, when applied to the various stock selections, still reaches its respective objective. We compare the attainment of the respective objective of the diversified portfolios of selected stocks with the broad cap-weighted and cap-weighted portfolios of selected stocks.

From Panels A, B, and C of Exhibit 9, it is interesting to observe that the diversification-based weighting strategies when applied to stock selections attain the respective objective better than the broad cap-weighted portfolios. This is true for all three diversified weighting schemes across the nine stock selections, with the notable exception of the minimum-volatility weighting applied to high-volatility stocks, which unsurprisingly does not allow for lowering volatility compared with the broad cap-weighted index. However, for a fair assessment of the performance of the weighting schemes, one should compare the attainment of objectives of these strategies with the cap-weighted portfolio for the respective stock selection. The diversification-based strategies outperform their cap-weighted counterparts in the same selection universe with no exception, providing strong evidence that choosing the right weighting scheme to attain a given objective is effective whatever the stock selection.

Overall, the empirical results for a wide variety of advanced beta benchmarks that flexibly draw on different specifications for the constituent selection and the weighting scheme show that a key difference between a stock selection approach and a diversification-

Exhibit 7
Stock Selection Approaches

The exhibit defines various stock characteristics that are used to sort the stocks and to divide the stock universe into explicitly factor tilted groups.

Variable for Stock Selection	Resulting Portfolios		
Size (market cap)	Small	Medium	Large
Total Volatility	Low Vol (Defensive)	Medium Vol	High Vol (Aggressive)
Dividend Yield	Growth	Blend	Value

Exhibit 8

Controlling Factor Exposures through Stock Selection

The table shows the excess (over S&P 500) risk factor exposures of the minimum-volatility, efficient maximum-Sharpe Ratio, and maximum-decorrelation portfolios based on the broad S&P 500 stock universe and nine different stock selections. The stock selection is defined at each rebalancing and takes into account the past two years' weekly data. Weekly total return data from July 5, 1963–December 31, 2010, are used for the analysis, and values significant at the 1% level are highlighted in bold. We run the following regressions to identify factor exposures

$$R_P - R_{CW} = \alpha + \beta_M \cdot R_{CW} \quad (1)$$

$$Res = \beta_F \cdot R_F \quad (2)$$

R_P is the time series of test portfolio returns, R_{CW} is the S&P 500 time series returns, β_M is the market beta, β_F is the risk factor beta, and R*es* is the residual time series from the Equation (2) regression. This two-step process is used for each risk factor and test portfolio. The bold values indicate that the beta for the factor tilt is significant at the 1% confidence level.

Panel A	Minimum Volatility Portfolios									
		Size-based stock selection			Volatility-based stock selection			Dividend Yield-based stock selection		
	All	Small	Medium	Large	Low Vol	Medium	High Vol	Growth	Blend	Value
Market	**–26.20%**	**–25.41%**	**–28.03%**	**–23.92%**	**–36.36%**	**–15.06%**	–1.35%	**–16.12%**	**–27.22%**	**–30.87%**
Size (Big-Small)	**–19.00%**	**–43.75%**	**–19.32%**	1.83%	**–12.17%**	**–26.73%**	**–47.77%**	**–13.96%**	**–15.52%**	**–25.59%**
Volatility (High-Low)	**–4.94%**	2.83%	**–6.27%**	**–10.34%**	**–11.80%**	–0.24%	**15.21%**	–1.36%	**–7.43%**	**–5.93%**
Div Yield (High-Low)	**13.28%**	**16.92%**	**15.40%**	**10.31%**	**18.52%**	**13.19%**	**8.42%**	**–4.26%**	**13.33%**	**25.47%**
Panel B	**Efficient Maximum–Sharpe Ratio Portfolios**									
Market	**–21.92%**	**–23.69%**	**–23.94%**	**–20.09%**	**–33.86%**	**–11.38%**	4.90%	**–11.89%**	**–23.68%**	**–27.46%**
Size (Big-Small)	**–21.13%**	**–46.28%**	**–21.40%**	0.29%	**–12.05%**	**–27.03%**	**–50.13%**	**–16.31%**	**–17.82%**	**–30.74%**
Volatility (High-Low)	–2.10%	**4.63%**	**–3.76%**	**–7.25%**	**–11.50%**	0.41%	**21.01%**	**3.38%**	**–5.61%**	–3.31%
Div Yield (High-Low)	**14.36%**	**17.89%**	**16.04%**	**9.76%**	**18.30%**	**14.26%**	6.28%	**–8.69%**	**14.81%**	**30.91%**
Panel C	**Maximum–Decorrelation Portfolios**									
Market	**–8.60%**	**–7.93%**	**–10.57%**	**–6.59%**	**–29.53%**	**–11.72%**	**8.24%**	–0.82%	**–14.03%**	**–12.54%**
Size (Big-Small)	**–37.07%**	**–65.59%**	**–27.26%**	**–3.15%**	**–14.70%**	**–28.67%**	**–59.28%**	**–33.19%**	**–30.53%**	**–49.97%**
Volatility (High-Low)	**10.28%**	**19.43%**	**6.16%**	1.36%	**–9.75%**	1.24%	**27.31%**	**15.33%**	**3.59%**	**8.28%**
Div Yield (High-Low)	**8.59%**	**14.23%**	**6.95%**	1.25%	**16.76%**	**13.82%**	4.47%	**–16.66%**	**11.80%**	**36.26%**

based weighting scheme is that the latter allows for taking into account the interaction effects that exist between stocks and hence leads to attaining the objective more effectively. An issue with simple stock selection is that, to attain a given objective, one often needs to exclude a majority of stocks and accept the resulting concentration. However, rather than seeing stock selection and diversification-based weighting as competing choices,

EXHIBIT 9

Attaining the Diversification Objective Compared with Cap-Weighted Portfolios for Various Stock Selections

The exhibit compares the attainment of the diversification objective of the three diversification-based portfolios: minimum volatility, efficient maximum Sharpe ratio, and maximum decorrelation, each based on the broad S&P 500 universe and nine different stock selections. The objective is compared against the S&P 500 index as well as against the cap-weighted index on the same stock selection as the diversification-based strategy. The improvement in respective objective refers to the relative decrease in volatility attained by the GMV-NC portfolio, the relative increase in Sharpe ratio attained by the efficient MSR portfolio, and the relative decrease in the GLR concentration measure attained by the MDC portfolio. Weekly total return data from January 2, 1959–December 31, 2010, are used for the analysis.

Panel A	Minimum-Volatility Weighting with Norm Constraints									
		Size-based stock selection			Volatility-based stock selection			Dividend Yield-based stock selection		
Reduction in Volatility	**All**	**Small**	**Medium**	**Large**	**Low Vol**	**Medium**	**High Vol**	**Growth**	**Blend**	**Value**
% Reduction relative to broad CW	19.8%	11.6%	18.0%	18.6%	26.6%	6.4%	–12.3%	8.8%	18.6%	19.7%
% Reduction relative to CW portfolio with same stock selection	19.8%	24.0%	21.6%	19.3%	17.3%	15.3%	24.4%	23.7%	15.8%	19.4%
Panel B	**Efficient Maximum–Sharpe Ratio Portfolios**									
Increase in Sharpe Ratio										
% Increase relative to broad CW	85.6%	139.9%	85.9%	30.2%	98.5%	71.0%	42.4%	103.0%	99.1%	39.8%
Increase relative to % CW portfolio with same stock selection	85.6%	57.8%	23.6%	55.4%	64.8%	64.5%	251.6%	90.6%	129.8%	114.9%
Panel C	**Maximum–Decorrelation Portfolios**									
Reduction in GLR Concentration Measure										
% Reduction relative to broad CW	43.1%	45.3%	31.9%	15.0%	22.4%	27.4%	39.5%	34.5%	32.6%	38.9%
% Reduction relative to CW portfolio with same stock selection	43.1%	25.5%	19.5%	25.6%	36.3%	35.2%	42.5%	45.5%	38.3%	36.9%

additional possibilities arise when combining them to reach an objective while controlling the tilt toward certain stock characteristics. In particular, stock selection is an effective means of tilting a portfolio toward the desired risk factor exposures or for neutralizing implicit factor exposure decisions brought about by a weighting scheme.

BENCHMARKING POPULAR ADVANCED BETA EQUITY METHODOLOGIES

The advanced beta benchmarks analyzed in the previous section have the advantage of providing a consistent and flexible framework to make construction choices for alternative equity index strategies. In this section, we draw on this benchmark construction framework to assess commercially available strategies. More specifically, for a given commercial index, it is possible to construct advanced beta benchmarks that pursue similar objectives and have similar risk factor exposures or similar constraints but are built on a state-of-the-art methodology of diversification. Since our benchmarks distinguish between the different phases of index construction, we are able to provide a transparent assessment of different methodology components. Such benchmarks are suitable references for commercial advanced beta strategies in that they allow for comparing results and providing insights into additional strategy specifications that may be appropriate for the respective investment objective. It should be noted that the time periods used for the analysis below are conditioned by the availability of index data for the respective index on Bloomberg.

To illustrate such benchmarking, we select two types of commercial strategies that have received considerable attention, namely, defensive equity strategies and fundamental equity indexation strategies.

Analyzing Defensive Equity Strategies

A wide variety of defensive strategies exist, and it is clear that any comparison across them would lead to results that would be difficult to interpret given that such strategies may vary with respect to numerous methodological and implementation choices. Below, we analyze three popular defensive indices—the S&P 500 Low Volatility Index, Russell Defensive Index, and MSCI Minimum Volatility Index—by constructing advanced beta benchmarks that have a similar low-risk objective and similar constraints as the commercial index.

Benchmarking the S&P Low Volatility Index: choosing a weighting scheme for a low volatility selection. The S&P 500 Low Volatility Index consists of a stock selection of the 100 least-volatile stocks that are then weighted by the inverse of their volatility (also see Exhibit 2). Exhibit 10 shows the results for the published index and compares them to various advanced beta benchmarks. In particular, if one accepts the quite aggressive stock selection of including only 20% of stocks in the standard index universe, the question of a suitable weighting scheme arises. The advanced beta benchmarks in Exhibit 10 thus provide an overview of the results obtained with an extensive set of possible weighting schemes, which may be suitable alternatives to inverse-volatility weighting.

The results show that the S&P Low Volatility Index slightly lowers the volatility over our benchmarks that cap weight or equal weight the 100 least volatile stocks. We also construct benchmarks that draw on diversification-based weighting schemes to exploit the correlation properties within this universe of 100 stocks. Exhibit 10 shows that all three diversification approaches (GMV-NC, efficient MSR, and MDC) lead to lower volatility than the S&P Low Volatility Index while also improving their respective diversification objective. These results thus lead to a clear recognition that once the selection decision of the index has been made, natural alternatives to the chosen weighting scheme exist that can appropriately address both the low-volatility objective and other relevant objectives. For example, MDC weighting leads to slightly lower volatility than the commercial index and at the same time improves the decorrelation across the reduced set of stocks.

Benchmarking the Russell Defensive Index: achieving high liquidity and low volatility. Another example of a bundled defensive equity strategy is the Russell Defensive Index, which tries to achieve low volatility through a stock selection and then cap weights its constituents. This choice for cap weighting could be justified by an objective of liquidity, and Russell indeed refers to the "high investment capacity"[20] of its approach.[21] However, it is reasonable to ask what the alternatives are to this choice. In particular, one may pursue the objective of low portfolio volatility through a diversification-based weighting scheme as opposed to through a stock selection strategy. Indeed, Exhibit 3 has shown that minimum-volatility weighting allows for a higher degree of volatility reduction than a low-volatility stock selection strategy. However, in the presence of strong liquidity constraints, cap weighting makes obvious sense, as it not only ensures a high average market capitalization within the index but actually makes sure that there will be no capacity issues in any stock when investors are moving in or out of the index.[22]

When relying on a portfolio optimization, such capacity constraints may obviously be exceeded. However, a straightforward way of addressing such capacity constraints is to use liquidity rules that impose an upper bound on the relative weight with respect to the market-cap weight of the stock in the broad index. We create an advanced beta benchmark that weights stocks based on

EXHIBIT 10

Benchmarking the S&P 500 Low Volatility Index

The exhibit presents the performance statistics, deconcentration measures, and relative risk statistics of the portfolios based on the low-volatility selection (bottom-100 volatile stocks in the S&P 500 universe, like the S&P 500 Low Volatility Index methodology) against the S&P 500 benchmark. Published S&P 500 Low Volatility Index, cap-weighted, inverse-volatility-weighted, equal-weighted and optimized norm-constrained minimum-volatility, maximum-Sharpe Ratio, and maximum-decorrelation portfolios are analyzed. All statistics are annualized, and performance ratios that involve the average returns are based on the geometric average. Due to the short history of the Bloomberg return series, weekly total return data from November 16, 1990–December 31, 2010, is used for the analysis. (*) requires information on index weights. The improvement in respective diversification objective refers to the relative decrease in volatility attained by the GMV-NC portfolio, relative increase in Sharpe ratio attained by the efficient MSR portfolio, and relative decrease in the GLR concentration measure attained by the MDC portfolio.

	All S&P 500 stocks	100 lowest-volatility stocks in the S&P 500 universe					
			Ad hoc Weighting		Diversified Weighting		
	Cap weighting (S&P 500)	S&P 500 Low Vol Index	Cap weighting	Equal weighting	GMV–NC	Efficient MSR	MDC
Ann Returns	9.46%	10.31%	9.15%	11.18%	10.42%	10.49%	10.81%
TE with S&P 500	0.00%	9.53%	9.07%	9.23%	10.73%	10.34%	9.86%
Eff. Number	125.02	N.A.(*)	28.82	100.00	33.33	39.42	43.33
Volatility	16.92%	12.95%	13.58%	13.15%	11.94%	12.18%	12.49%
Reduction in Volatility compared with S&P 500 Low Vol Index		–	–4.9%	–1.6%	7.7%	5.9%	3.5%
Improvement of respective Diversification Objective compared with S&P 500 Low Vol Index		–	–	–	7.7%	9.1%	N.A.(*)

the minimum-variance approach used in the remainder of this article; instead of using norm constraints, we impose a constraint that weights cannot exceed a multiple of their weight in the broad cap-weighted index. In contrast to the Russell Defensive Index, we draw on all stocks in the standard reference index rather than selecting stocks based on their defensiveness. Exhibit 11 shows the results obtained for this diversification-based weighting scheme with tight liquidity constraints and compares them to the Russell Defensive Index.

The results in Exhibit 11 confirm the conclusions of Exhibit 3, namely, that minimum-volatility weighting is able to add value to a pure stock selection approach when aiming at defensive equity exposure. In particular, the minimum-volatility optimization lowers volatility by about 6% with respect to the Russell Defensive Index. It should also be noted that the Russell Defensive Index draws not only on volatility, but also on other measures of defensiveness such as low leverage (see Exhibit 2), and therefore, it is not identical to a simple low-volatility stock selection. However, the results that our minimum-volatility benchmark obtains lower volatility than the Russell Defensive Index suggests, perhaps, that taking into account correlations through such a diversification scheme may be an issue of higher-order importance than multiplying the criteria used to assess defensiveness within a stock selection.

Benchmarking the MSCI Minimum Volatility Index: selection of weight constraints. We have benchmarked stock-selection or ad hoc-weighted defensive advanced beta strategies by analyzing results obtained when changing the choice of weighting scheme to minimum volatility. It is clear that the minimum-volatility weighting approach in the advanced beta benchmarks we constructed itself contains a number of methodological choices, such as the norm constraints and the covariance matrix estimation. Commercial minimum-volatility strategies likewise make a range of such choices of specification. It is interesting to analyze whether such methodological differences lead to differences in performance and risk.

Exhibit 11

Benchmarking the Russell Defensive Index: Weighting vs. Selection to Achieve Low Volatility under a Strong Liquidity Constraint

The exhibit presents the performance statistics, deconcentration measures, and relative risk statistics of published MSCI Russell 1000 Defensive Index and an optimized minimum-volatility portfolio of S&P 500 stocks with individual security weight constraints, against the S&P 500 cap-weighted benchmark. All statistics are annualized, and performance ratios that involve the average returns are based on the geometric average. Due to the short history of the Bloomberg return series, weekly total return data from July 5, 1996–December 31, 2010, are used for the analysis.

	Cap Weighting (S&P 500)	Russell 1000 Defensive Index	Minimum Volatility with Tight Liquidity Constraints
Annualized Returns	6.60%	6.94%	7.03%
Volatility	18.93%	16.16%	15.18%
TE with S&P 500	0.00%	5.35%	6.60%
Eff. Number	117.4	–	64.8
Reduction in Volatility compared with Russell 1000 Defensive Index			6.1%

In order to conduct such an analysis, we consider the MSCI Minimum Volatility Index and construct an advanced beta benchmark with similar objectives and constraints. The MSCI index uses a set of rigid weight constraints that specify upper and lower bounds for each constituent. The index also uses upper and lower bounds on sector weight differences with respect to the cap-weighted version of the same stock selection. Comparing such an index to the minimum-volatility advanced beta benchmark we used above would not account for this control of sector exposures.

For this reason, we derive a minimum-volatility benchmark that—like the MSCI Minimum Volatility Index—constrains sector weights to lie within ±5% of the sector allocation of the cap-weighted reference index. Except for this change—which is meant to apply not only the same objective as the commercial index, but also the same sector constraint to our advanced beta benchmark—our minimum-volatility approach is identical to the one used above. In particular, in addition to the sector constraints, instead of using rigid upper and lower bounds on each individual stock, we use the same norm constraint as in the GMV portfolios above. Exhibit 12 compares the risk and return properties of the MSCI Minimum Volatility Index with those of our benchmark.

It is clear from these results that when pursuing the same diversification objective (low volatility), using the same sector constraints, the advanced beta minimum-volatility benchmark adds value. In fact, volatility is reduced by about 11.6% relative to the MSCI Minimum Volatility Index. Moreover, the advanced beta minimum-volatility benchmark also leads to an improvement in the Sharpe ratio over the MSCI index. It should, however, be noted that our advanced beta benchmark also has a higher tracking error, which may be undesirable to investors. However, as the MSCI Minimum Volatility Index does not contain an explicit tracking-error objective, we have not considered such an objective in the advanced beta benchmark. Having said that, if one wanted to benchmark commercial strategies that do have an explicit tracking-error objective, it would be straightforward to include a suitable relative risk control mechanism in the benchmark to match the relevant tracking-error level. In particular, Amenc, Goltz, Lodh, and Martellini [2012] have shown that it is possible to control the average and extreme tracking error of minimum–variance benchmarks.

Disentangling Fundamental Stock Selection Decisions from Weighting Scheme Decisions

Another interesting illustration for how one can deconstruct a commercially available strategy and make explicit choices on various index construction steps is

EXHIBIT 12

Benchmarking the MSCI Minimum Volatility Index: Selection of Weight Constraints

The exhibit presents the performance statistics, deconcentration measures, and relative risk statistics of the published MSCI USA Minimum Volatility Index, optimized minimum-volatility portfolio with sector and individual security weight constraints, and optimized minimum-volatility portfolio with sector and norm weight constraints, against the S&P 500 benchmark. All statistics are annualized, and performance ratios that involve the average returns are based on the geometric average. Due to the short history of the Bloomberg return series, weekly total return data from January 1, 1999–December 31, 2010, are used for the analysis.

	Cap-Weighted S&P 500	MSCI USA Minimum Volatility Index	Minimum Volatility Benchmark with Sector Constraints and Flexible Norm Constraints
Annualized Returns	2.13%	3.43%	6.42%
Volatility	19.41%	16.27%	14.39%
TE with S&P 500	0.00%	6.53%	9.66%
Eff. Number	113.4	–	100.0
Reduction in Volatility compared to MSCI USA Minimum Volatility Index	–	–	11.6%

the case of fundamental equity indexation strategies. More specifically, we assess portfolios which select stocks by their composite fundamental size measure made up from revenues, cash flows, and book value. We select the top-500 stocks according to this fundamental size measure as an alternative to the standard stock selection in the S&P 500, which is based on the stock's market cap. As one can use any weighting scheme on such a fundamentals-based stock selection, we assess the risk and return obtained with various ad-hoc and diversification-based weighting schemes. It should be noted that, due to the availability of fundamental data from Worldscope, the time period of the analysis in this section is shortened to January 5, 1984–December 31, 2010.

One could, of course, argue that applying diversification-based weighting instead of fundamental weighting to a fundamental stock selection does not take into account the objective of representativeness, which fundamental equity indexation aims at. Clearly, while fundamental weights reflect a measure of relative firm size, the same cannot be said of weights resulting from a diversification scheme. However, another rationale brought forth by promoters of fundamental equity indexation is that such products avoid the most overpriced stocks (see Hsu [2006]). In fact, Arnott and Kuo [2011] argue that stock selection by fundamentals, by being indifferent to market price, can be expected to lead to higher performance than stock selection by market cap.[23] Our tests of diversification-weighted portfolios of fundamentally selected stocks try to exploit this rationale while avoiding the pitfall of insufficient diversification across the selected stocks.

Exhibit 13 shows that fundamental stock selection adds about 50 basis points (bps) to annual performance, compared with the standard S&P 500 portfolio, over the sample period when using cap weighting within the fundamental stock selection. When weighting by the same fundamental composite measure that has been used for stock selection, one increases annual outperformance by approximately another 100 bps. However, all other alternative weighting schemes we test (i.e., equal weighting, minimum-volatility weighting, efficient maximum-Sharpe ratio weighting, and maximum-decorrelation weighting) achieve higher levels of risk-adjusted returns (Sharpe ratio) when applied to the fundamental stock selection than fundamental weighting.

Moreover, an important limitation of the fundamental weighting scheme is that it is not clear what its objective is. The results in Exhibit 13 for the effective number of stocks suggest that fundamental weighting allows for deconcentrating the portfolio to some degree compared with using the market-cap-weighted version of the fundamentally selected stocks. The effective number of stocks increases to 153 from about 114 with cap weighting. This result is not surprising; a composite measure of fundamental firm size is built on different variables and hence should allow for some more decon-

EXHIBIT 13

Performance Statistics of Portfolios Using Fundamentals-Based Stock Selection

The table presents the performance statistics and deconcentration measures of the portfolios based on fundamental size selection. Cap-weighted, fundamental-weighted, equal-weighted and diversification-based minimum-volatility, efficient maximum-Sharpe ratio, and maximum-decorrelation portfolios with and without relative risk control are analyzed. For fundamental weighting, the last available fundamental is used to weight the stocks. Only year-end fundamental values are available, so the latest annual value for each stock is used. All statistics are annualized, and performance ratios that involve the average returns are based on the geometric average. The improvement in respective objective refers to the relative decrease in volatility attained by the GMV-NC portfolio, relative increase in Sharpe ratio attained by the efficient MSR portfolio, and relative decrease in the GLR concentration measure attained by the MDC portfolio. Weekly total return data from January 5, 1984–December 31, 2010, are used for the analysis.

	Selection by Mkt Cap	Selection by Fundamental Size (Book Value+Cash Flow+Revenues) – Top 500 stocks					
		Heuristic Weighting Strategies			Diversified Weighting Strategies		
	CW (S&P 500)	CW	Fundamental-Weighted	EW	GMV-NC	Efficient MSR	MDC
Ann Returns	10.56%	10.89%	12.05%	12.78%	12.22%	12.86%	13.99%
TE with S&P 500	0.00%	3.19%	4.81%	5.62%	7.30%	6.67%	8.80%
Eff Number	126.1	113.7	153.0	500.0	166.7	120.3	167.0
Ann Std Dev	16.69%	16.31%	16.57%	16.28%	13.20%	13.59%	17.67%
Sharpe Ratio	0.36	0.39	0.45	0.51	0.58	0.61	0.53
GLR Measure	0.239	0.223	0.194	0.164	0.159	0.162	0.125
Improvement of Objective over Fundamental Weighting through Diversification	-	-	-	-	21%	36%	36%

centration than relying on a single variable in the case of market cap.

For the diversification-based weighting schemes, however, there is a clearly defined objective. It is interesting to assess whether an investor who aims at a specific objective can obtain this objective within the fundamentally selected universe. Exhibit 13 below shows the degree to which minimum-volatility weighting, efficient maximum-Sharpe ratio weighting, and maximum-decorrelation weighting attain their objective.

Compared with the fundamental-weighted version of the fundamental stock selection, each weighting scheme improves the respective objective. This result is not surprising; the fundamental weighting and stock selection approaches, by failing to make a clear distinction between the two index construction steps, lead to forgetting that diversification may be an important source of value added. The relative improvements obtained by diversification-based weighting schemes in terms of attaining the objective are considerable in magnitude, with each objective being improved by at least 20%. For example, while the volatility of the fundamental-weighted portfolio of fundamentally selected stocks is 16.57%, minimizing the volatility within the universe of fundamentally selected stocks reduces volatility to 13.2%. Disentangling the weighting scheme from stock selection thus improves the attainment of a defined risk–return objective while specifying in which stocks, and hence which risk factor exposures, the investor wants to invest.

CONCLUSION

Alternative equity index strategies have seen widespread growth over the past years, but investors are at a loss when it comes to analyzing and understanding such strategies. Because providers offer prepackaged choices mixing decisions on separate steps of index construction and offers from the same provider do not follow a coherent framework of index construction, analyzing such strategies is indeed difficult. This article proposes a framework for constructing advanced beta benchmarks by disentangling the different steps of the construction process. By flexibly drawing on a variety of choices within this framework, we construct a wide range of strategies that help to assess the impact of different choices.

Our analysis shows that there is a key distinction to be made between a stock selection decision—which will help to tilt the portfolio toward the relevant characteristics—and the choice of a diversification scheme—which will define the strategy of combining the relevant stocks by taking into account how they interact within a portfolio. In particular, our empirical results suggest that a diversification-based weighting scheme is more effective in attaining the relevant diversification objective than a pure stock selection strategy. Moreover, our results show that, even if a stock selection is desired by an investor, forgetting about diversification benefits can lead to high opportunity costs, because diversification-based weighting schemes consistently allow for improving the risk–return trade-off for a given stock selection.

In fact, stock selection is a simple tool to correct the risk factor exposures of diversification-based weighting schemes by excluding stocks with the undesired characteristics prior to applying a diversification scheme. In this respect, it is interesting to note that diversification-based weighting schemes lead to considerable improvements in their respective objective, even after relevant factor exposures have been corrected. This result contradicts the claim that diversification-based weighting schemes boil down to simple factor tilts.

In addition to analyzing strategy construction in detail, the framework in this article can be used to construct strategies with objectives and constraints that are similar to commercial index strategies. Such strategies can then be used as benchmarks to assess the performance of commercial index offerings. This benchmarking approach is an important step beyond currently existing comparison studies. Such comparisons typically do not give much consideration to the relevant objective but rather try to figure out which strategy is superior to the others in terms of historical performance. The advanced beta benchmark approach in this article instead recognizes that different strategies may have different objectives. For a given objective, it is then interesting to compare and assess alternatives.

While such advanced beta benchmarks can provide a better understanding of the properties of individual strategies, a key question for investors who want to improve the performance of their portfolios over cap-weighted indices is how to successfully combine various advanced beta methodologies. This question points to several important issues for future research. One important consideration is that, fortunately, the question of combining strategies that aim to outperform cap-weighted indices is all but new, as it has been addressed in the context of allocating assets to multiple managers. There is an obvious analogy between advanced beta indices and active management, which is underscored by the fact that providers with a long history of analyzing manager styles have been launching advanced beta equity indices that mimic particular active management approaches.[24]

Therefore, similar to a situation for which one uses active managers, adopting advanced beta equity strategies—and thus choices in terms of weighting scheme and stock selection—leads to two types of risk: systematic risk factor exposures and strategy-specific risk. The strategy-specific risk may also be referred to as model risk. In the case of active managers, this risk is related to the manager's investment process and its evaluation of macro- and microeconomic conditions. A key difference between active managers and advanced beta indices is that the decision-making processes of index strategies are more systematic and rule based; hence, it is easier to document the strategy-specific risk. When defining combinations of advanced beta equity strategies, one should aim at taking into account not only the systematic risk factor exposures, but also the specific risk of the respective advanced beta equity strategy.

Another important issue relating to the combination of strategies is that of their conditional performance and risk. While such strategies result in unconditional risk factor tilts and performance improvements over cap-weighted indices, an investor may be interested in the conditional properties of such strategies. Amenc et al. [2012], for example, have shown that minimum-volatility and efficient maximum-Sharpe ratio strategies perform differently in bull and bear markets and in high- and low-volatility regimes. They use this insight to construct a diversified weighting scheme that combines both strategies. Alternatively, investors may want to combine strategies not only to diversify, but also to potentially time the market across strategies to benefit from predictions of market conditions. Whether for diversification or timing decisions, an analysis of the conditional risk and performance properties provides useful insights. In that respect, both the systematic component of the returns of advanced beta strategies and the strategy-specific component could clearly display conditional patterns that investors may want to exploit.

APPENDIX

CONSTRUCTION OF EQUITY RISK FACTORS

In order to derive explicit factors, we use a set of characteristics to create long/short (high minus low) portfolios, and the returns of these portfolios are interpreted as factors. All NYSE, AMEX, and (after 1972) NASDAQ stocks are used to form the factors, and all factors are rebalanced quarterly. Weekly data from July 5, 1963–December 31, 2010, are obtained from CRSP for this exercise, in a manner similar to Fama and French [1992]. Every quarter, we sort stocks into five quintiles based on certain characteristics for the past two years. We then create a cap-weighted high portfolio of stocks in the top quintile and a cap-weighted low portfolio of stocks in the bottom quintile. The difference between the returns of these two portfolios is the factor return. The characteristics are:

Size factor: Market cap at the time of rebalancing is used to sort the stocks. The size factor is large (big–small), as opposed to the more commonly used small (small–big) Fama–French factor.

Volatility factor: Stocks are sorted by their volatilities, which are computed by using weekly returns over the past two years.

Dividend-Yield factor: Average past two-year dividends and current price (at the time of rebalancing) are used to compute the dividend yield for each stock. These stocks are then sorted based on their dividend yield.

ENDNOTES

[1]While the article shows results for a single factor regression, in particular the alpha of the alternative index with respect to a single market factor, and provides a detailed discussion of these results, the article only loosely refers to the existence of small-cap and value exposure, without showing any results to the reader.

[2]There is a consensus today that comparing—for example, fundamental equity indexation, which leads to a small-cap and value tilt, with cap-weighted indices, which have a large-cap and momentum tilt—without taking into account the differences in factor exposures is not particularly sensible. Moreover, a concern over comparability among advanced beta strategies has led certain authors to document the risk of deviations from cap weighting in terms of average and extreme tracking error risk (see Amenc et al. [2012]) and to propose methodologies to control this risk (see Amenc and Retkowsky [2011] or Martellini and Milhau [2012]).

[3]It is useful to outline the differences of our approach from the article by Chow et al. [2011], which analyzes the performance of their replication of commercially available indices. Such a replication approach is exposed to a risk of diverging in important aspects from the concept actually implemented in practice. Because the replication is not systematic, which parts of the methodology will diverge is not entirely transparent and is up to the discretion of the author. Furthermore, by analyzing such self-constructed bundles of methodological decisions, nothing has been gained in terms of understanding how the different parts of the methodology contribute to the overall performance and risk. For example, when Chow et al. compare minimum-volatility indices that change only the weighting scheme of standard index constituents with fundamentals-based indices that rely on both stock selection and alternative weighting, one is effectively comparing apples to oranges and does not obtain a better under standing of what drives the performance of these strategies.

[4]The fact that commercial indices rely on a mix of choices on different steps in the index construction process has the implication that the overall performance and risk of the index can no longer be attributed to a single decision. Whether it is the stock selection or weighting decision that drives the performance difference with standard cap-weighted indices is often unclear because the objective and role of the different methodological steps is often not clearly identified. Decomposing the methodology and generating benchmarks reflecting the different steps of the methodology provides a means to attribute the performance of an index to the choices made in the different steps.

[5]More precisely, when using minimum-volatility weighting within a fundamentals-based stock selection, one lowers volatility by 20% compared with using fundamentals-based weighting. Likewise, when using efficient maximum-Sharpe ratio weighting, one increases the Sharpe ratio by 36%, and when using maximum-decorrelation weighting, one reduces the Goetzmann–Li–Rouwenhorst concentration measure by 37%.

[6]See, for example, Martellini [2011] for a derivation of the conditions of optimality of equal weighting and DeMiguel, Garlappi, and Uppal [2009] for an extensive empirical assessment of performance of equal-weighted portfolios.

[7]The effective number of stocks is defined as the reciprocal of the Herfindahl Index, a standard measure of portfolio concentration that is given by the sum of squares of portfolio weights.

[8]One could argue that because such weighting schemes do not use a theoretical framework such as modern portfolio theory, they remain ad hoc weighting schemes. Any strategy that

does not have a sound theoretical justification will be exposed to the high risk of suffering from data-snooping biases.

[9]The index also considers other measures of defensiveness in addition to volatility.

[10]Similar approaches are being developed by other providers of fundamental equity indexation products. Although fundamental equity indexation products were launched following the bursting of the technology bubble as a strategy that would have been able to avoid exposure to this bubble, such indices were not able to avoid the banking and sovereign crisis in the 2008–2011 period and have actually seen poor performance over this period. Providers of fundamentals-based equity indices have since been looking for introducing a more defensive or low-volatility component to their strategies.

[11]While characteristics-based weightings are similar to stock selection in the sense that they ignore diversification, it should be noted that they are different from a pure stock selection in the sense that they keep positive weights for all stocks in the universe. They can therefore be seen as a way of tilting portfolios toward certain characteristics while maintaining a higher level of deconcentration among stocks in the universe than a pure stock selection. In particular, our overview in Exhibit 2 shows that it is common among index providers to use multiple variables in such ad hoc weighting schemes, which typically reduces concentration compared with weighting by a single variable.

[12]It has been shown that a pure volatility minimization weighting scheme often leads to strong concentration in particular sectors and that such concentration can have negative consequences in terms of extreme risk (see Chan, Karceski, and Lakonishok [1999] and Andersen, Malavergne, and Simonnetti [2000]).

[13]It is clear that such diversification approaches are only approximations of the true optimal portfolios, as such approaches will not perfectly reach their optimization objective out of sample, due to parameter estimation errors. It should also be noted that we impose long-only constraints for all portfolio optimizations.

[14]Consistent with what we discussed above, we do not include any deconcentration scheme based on attributing equal weights, because such ad hoc approaches by construction do not have any explicit objective in terms of the risk–return properties they would be supposed to improve. However, because it does not integrate any information on differences across stocks, equal weighting will be used as a neutral weighting method for portfolios that rely on stock selection, so as to be able to compare the results of diversification-based weighting schemes with those of a sole stock selection.

[15]The Herfindahl index, given by the sum of squares of portfolio weights, corresponds to the so-called 2-norm. We set the lower bound for the effective number of stocks at a level of at least N/3, where N is the number of constituents in the relevant universe. Our lower bound on the effective number of stocks corresponds to an upper bound on the 2-norm.

[16]More precisely, we start with the complete universe and compute each stock's sample correlation with an equal-weighted portfolio of remaining stocks. We sort the stocks by their correlations and drop the most correlated stock. We repeat this iterative procedure with this reduced universe and continue to drop stocks until the desired number (N) is reached.

[17]Both the stock selections and the optimizations are based on input parameters computed over the past 104 weeks of returns. All diversification-based strategies draw on a robust covariance matrix that is estimated using a statistical factor model, which follows the approach in Amenc et al. [2011]. All portfolios in this article are long only and are rebalanced quarterly, in coherence with the actual rebalancing dates of the S&P 500.

[18]Our results for the same tests applied to the Sharpe ratio objective show that none of the stock selection portfolios, whatever the level of concentration, is able to reach the Sharpe ratio of the optimization-based approach. The results for the decorrelation objective suggest that an increase in concentration is necessary to obtain similar levels of GLR concentration as with the maximum decorrelation approach.

[19]It should be noted that an alternative to using stock selection to correct factor exposures would be to include constraints on such factor exposures in the portfolio optimization used for a diversification-based weighting scheme. An advantage of using stock selection in a first step is that it provides for a perhaps more intuitive way of accounting for factor exposures than additional constraints in an optimization program. Moreover, it is clear that popular advanced beta products from index providers typically apply stock selection rather than factor constraints within an optimization program. Our approach of using stock selection to adjust for factor exposures is thus useful to understand the construction of such indices.

[20]See Thurston [2011].

[21]Of course, other arguments in favor of cap weighting are low turnover and simplicity.

[22]In our minimum-volatility weighting of all stocks in the index universe, we impose a constraint that the weight of each stock relative to its market-cap weight cannot exceed a multiple of two. This corresponds approximately to the result obtained when selecting half the stocks in the index and cap weighting them.

[23]It should, however, be noted that the finding of an outperformance of a fundamental selection over a standard index selection may be highly sample dependent, and the theoretical argument of fundamentals-based indexing being able to avoid the most overpriced stocks has been shown to be "fundamentally flawed" (Perold [2007] and Graham [2011]).

[24]An example is Russell Investments, which has launched indices for "Investment Disciplines," including Aggressive Growth, Consistent Growth, Growth-at-a-Reasonable-Price, Equity Income, Low P/E, and Contrarian. See Christopherson [2011] for a description of the origins of this index offering.

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